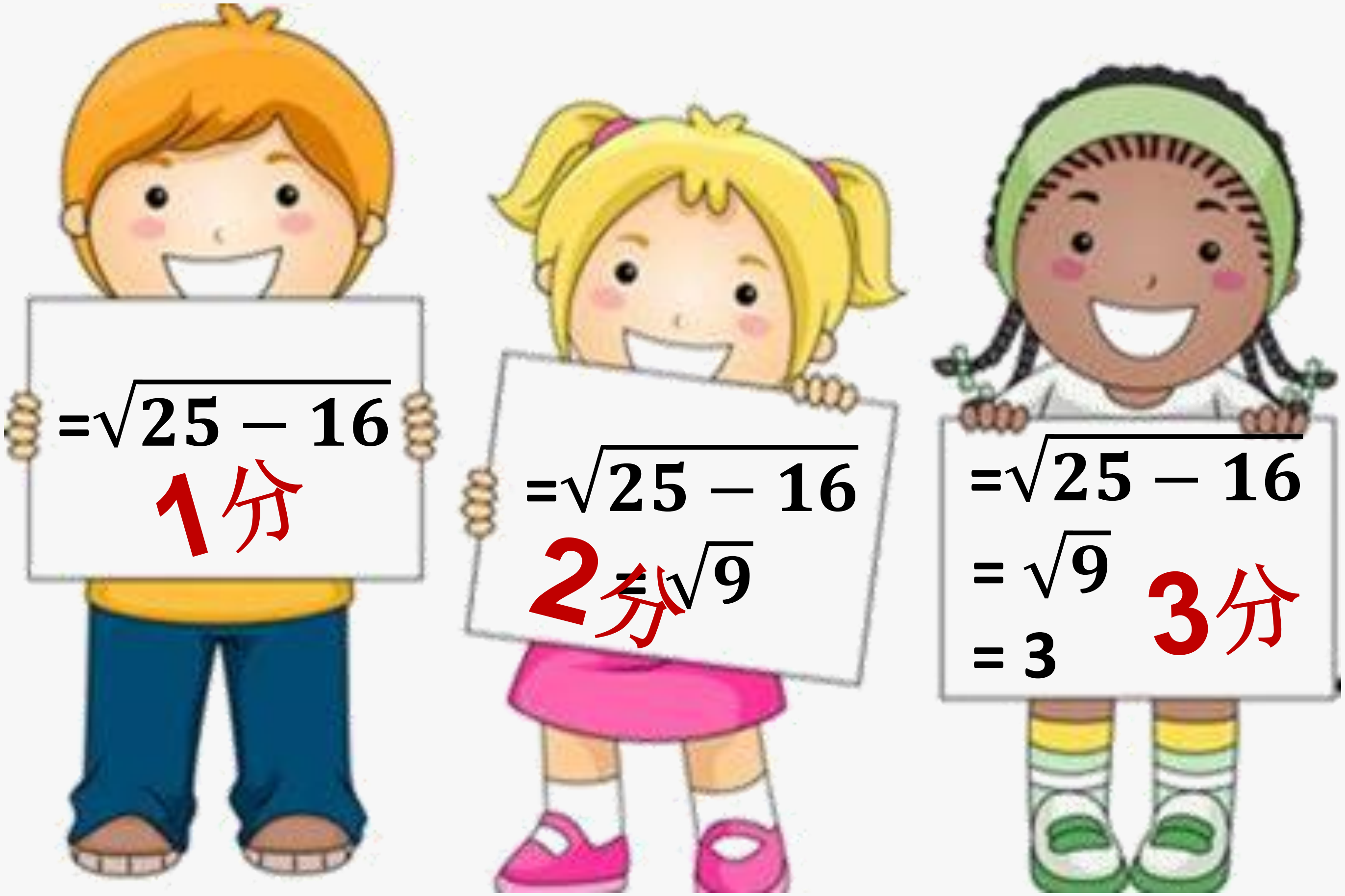
$$\sqrt{7.5/0.3-16}=?$$



A sequential cognitive diagnosis model for polytomous responses

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06 October 2016
IF: 1.93

The British Psychological Society
Promoting excellence in psychology

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Introduction

How to classify examinees into different latent classes with unique attribute patterns indicating mastery or non-mastery of a number of skills or attributes ?

↓

Cognitive diagnosis models (CDMs)

- deterministic inputs, noisy 'AND' gate, DINA
- deterministic inputs, noisy 'OR' gate, DINO
- generalized DINA, G-DINA
- log-linear CDM, LCDM
- general diagnostic model, GDM

Introduction

How to classify examinees into different latent classes with unique attribute patterns indicating mastery or non-mastery of a number of skills or attributes ?

Cognitive diagnosis models (CDM

- deterministic inputs
- determ ... DINO
- ... DINA
- ... CDM, LCDM
- general diagnostic model, GDM

for dichotomous responses

Introduction

How to deal with polytomously scored items?

- **Dichotomize**
  - **Using existing dichotomous CDMs**

loss of information

- **Polytomous CDMs**
  - **Partial credit DINA, PC-DINA**
  - **GDM for graded respon**
  - **nominal response diagn**
  - **polytomous LCDM**

overlook the relation between attributes and categories

Attribute and category association

- Solving an item consists of a finite number of sequential steps, each of which involves some attributes
- Score according to how many successive steps they have successfully performed
- Responses to items with h steps have h + 1 ordered categories

Attribute and category association

- Solving an item consists of a finite number of sequential steps, each of which involves some attributes
- Score according to how many successive steps they have successfully performe
- Responses to items with h s categories

$$\sqrt{7.5/0.3-16}$$

$=\sqrt{25-16}$ (division)

$=\sqrt{9}$ (subtraction)

$= 3$ (extraction of a root)

Attribute and category association

- traditional Q-matrix

  - a $J \times K$ binary matrix specifying whether an attribute is measured by an item

	Attributes			
Items	A_1	A_2	…	A_K
1	0	0	…	1
2	1	1	…	0
…	…	…	…	…
J	0	1	…	1

→ q_{jk}=1 / 0

Attribute and category association

- category-level Q-matrix (Q_C-matrix)

  - a $\sum_{j=1}^{J} H_j \times K$ binary matrix, which subscript C is used to denote category

Category	Attributes A_1	A_2	…	A_K
1	1	0	…	0
2	0	1	…	0
…	…	…	…	…
H	0	0	…	1

Attribute and category association

- category-level Q-matrix (Q_C-matrix)
  - an example: the restricted Q_C-matrix
  - the attribute and category association must be known a priori

Step	Category	Attributes: A_1 (division)	A_2 (subtraction)	A_K (extraction of a root)
$\sqrt{25-16}$	1	1	0	0
$\sqrt{9}$	2	0	1	0
3	3	0	0	1

e.g., after examinees have already achieved category 1, only the subtraction attribute is needed to perform category 2

Attribute and category association

- category-level Q-matrix (Q_C-matrix)
  - an example: the unrestricted Q_C-matrix
  - all attributes required by an item are needed by each category

Step	Category	Attributes A_1 (division)	A_2 (subtraction)	A_K (extraction of a root)
$\sqrt{25-16}$	1	1	1	1
$\sqrt{9}$	2	1	1	1
3	3	1	1	1

Sequential process model

- category-level Q-matrix : $\sum_{j=1}^{J} H_j \times K$ binary matrix
- the K binary attributes lead to 2^K latent classes with unique attribute patterns (i.e., $\alpha_c = (\alpha_{c1},\ldots, \alpha_{cK})$), where $c = 1,\ldots, 2^K$

  $\alpha_{ck}=1 \rightarrow$ mastered
  $\alpha_{ck}=0 \rightarrow$ not mastered

↓ sequential processing

$$s_{jh}(\alpha_c) = P\left(X_{ij} \geq h \mid X_{ij} \geq h-1, \alpha_c\right) = \frac{P\left(X_{ij} \geq h \mid \alpha_c\right)}{P\left(X_{ij} \geq h-1 \mid \alpha_c\right)}$$

Sequential process model

- the category response function for item *j*:

$$P(X_j = h|\boldsymbol{\alpha}_c) = [1 - S_j(h+1|\boldsymbol{\alpha}_c)] \prod_{x=0}^{h} S_j(x|\boldsymbol{\alpha}_c)$$

answer category *h* + 1 incorrectly

independent

categories 1, ..., *h* correctly

- $$\begin{cases} \sum_{h=0}^{H_j} P(X_j = h|\boldsymbol{\alpha}_c) = 1 \quad \forall c \\ S_j(h|\boldsymbol{\alpha}_c) = \begin{cases} 1, & \text{if } h = 0 \\ 0, & \text{if } h = H_j + 1 \end{cases} \end{cases}$$

Sequential G-DINA model

- partition the latent classes 2^K into 2^{K^*j} latent groups

$$K_j^* = \sum_{k=1}^{K} q_{jk}$$

- for category h: further collapse 2^{K^*j} latent groups into 2^{K^*jh}

the first K^*_{jh} attributes are required for category *h* of item *j*

$$\alpha_{ljh}^* = \left[\alpha_{l1}, \cdots, \alpha_{lk}, \cdots, \alpha_{lK_{jh}^*}\right]$$

$$l = 1, \ldots, 2^{K_{jh}^*}$$

Sequential G-DINA model

- using the identity link G-DINA model:

$$S_j(h|\boldsymbol{\alpha}_c) \Rightarrow S_j(h|\boldsymbol{\alpha}^*_{ljh})$$

$$S_j(h|\boldsymbol{\alpha}^*_{ljh}) = \phi_{jh0} + \sum_{k=1}^{K^*_{jh}} \phi_{jhk}\alpha_{lk} + \sum_{k'=k+1}^{K^*_{jh}} \sum_{k=1}^{K^*_{jh}-1} \phi_{jhkk'}\alpha_{lk}\alpha_{lk'} + \ldots + \phi_{jh12\ldots K^*_{jh}} \prod_{k=1}^{K^*_{jh}} \alpha_{lk},$$

intercept: ϕ_{jh0}

main effect due to α_{lk}: ϕ_{jhk}

two-way interaction effect due to α_{lk} and $\alpha_{lk'}$: $\phi_{jhkk'}$

K^*_{jh} way interaction effect due to α_{lk} to $\alpha_{lK^*_{jh}}$: $\phi_{jh12\ldots K^*_{jh}}$

Sequential G-DINA model

- using the identity link G-DINA model:

$$S_j(h|\boldsymbol{\alpha}_c) \longrightarrow S_j(h|\boldsymbol{\alpha}^*_{ljh})$$

$$S_j(h|\boldsymbol{\alpha}^*_{ljh}) = \phi_{jh0} + \sum_{k=1}^{K^*_{jh}} \phi_{jhk}\alpha_{lk} + \sum_{k'=k+1}^{K^*_{jh}} \sum_{k=1}^{K^*_{jh}-1} \phi_{jhkk'}\alpha_{lk}\alpha_{lk'} + \dots + \phi_{jh12\dots K^*_{jh}} \prod_{k=1}^{K^*_{jh}} \alpha_{lk},$$

$$S_j(h|\boldsymbol{\alpha}^*_{ljh}) \longrightarrow \{S_j(h|\boldsymbol{\alpha}^*_{ljh})\} \longrightarrow \boldsymbol{M}_{jh}\boldsymbol{\phi}_{jh}$$

$\phi_{jh} = \{\phi_{jh0}, \phi_{jh1}, \dots, \phi_{jh12\dots K^*_{jh}}\}$

invertible design matrix of dimension $2^{K^*jh} \times 2^{K^*jh}$

Sequential G-DINA model

- using the identity link G-DINA model:

$$S_j(h|\boldsymbol{\alpha}_c) \longrightarrow S_j(h|\boldsymbol{\alpha}_{ljh}^*)$$

$$S_j(h|\boldsymbol{\alpha}_{ljh}^*) = \phi_{jh0} + \sum_{k=1}^{K_{jh}^*} \phi_{jhk}\alpha_{lk} + \sum_{k'=k+1}^{K_{jh}^*} \sum_{k=1}^{K_{jh}^*-1} \phi_{jhkk'}\alpha_{lk}\alpha_{lk'} + \ldots + \phi_{jh12\ldots K_{jh}^*} \prod_{k=1}^{K_{jh}^*} \alpha_{lk},$$

$$S_j(h|\boldsymbol{\alpha}_{ljh}^*) \longrightarrow \{S_j(h|\boldsymbol{\alpha}_{ljh}^*)\} \longrightarrow \boldsymbol{M}_{jh}\boldsymbol{\phi}_{jh}$$

→ **RS-GDINA** (restricted)

→ **US-GDINA** (unrestricted)

Parameter estimation

- the conditional probability of the response vector $\mathbf{X}_i$ (i = 1,…,N)

$$P(\mathbf{X}_i|\boldsymbol{\alpha}^*_{lj}) = \prod_{j=1}^{J}\prod_{h=0}^{H_j} P(X_j = h|\boldsymbol{\alpha}^*_{lj})^{I(X_{ij}=h)}$$

reduced attribute pattern for the *l*th collapsed latent group for item *j*

an indicator variable evaluating whether X_{ij} is equal to h

Parameter estimation

- the E-step

- the expected number of examinees with attribute pattern α^*_{lj} scoring in category h

$$\bar{r}_{ljh} = \sum_{i=1}^{N} I(X_{ij} = h) P(\boldsymbol{\alpha}^*_{lj} | \mathbf{X}_i)$$

$$P(\boldsymbol{\alpha}^*_{lj} | \mathbf{X}_i) = \frac{P(\mathbf{X}_i | \boldsymbol{\alpha}^*_{lj}) p(\boldsymbol{\alpha}^*_{lj})}{\sum_{l=1}^{2^{K^*_j}} P(\mathbf{X}_i | \boldsymbol{\alpha}^*_{lj}) p(\boldsymbol{\alpha}^*_{lj})}$$

Parameter estimation

- the M-step

  - maximize the objective function with respect to item parameters φ_j

$$\ell = \sum_{l=1}^{2^{K_j^*}} \sum_{h=0}^{H_j} \bar{r}_{ljh} \log \left[\hat{P}(X_j = h | \boldsymbol{\alpha}_{lj}^*) \right]$$

optimization

- the Nelder and Mead (1965) simplex method

  - after generating a geometric simplex, its convergence is guided by moving the simplex appropriately

Parameter estimation

the expectation of category response function for item *j*

$$P(X_j = h|\boldsymbol{\alpha}_c) = [1 - S_j(h+1|\boldsymbol{\alpha}_c)] \prod_{x=0}^{h} S_j(x|\boldsymbol{\alpha}_c)$$

$$\hat{P}(X_j = h|\boldsymbol{\alpha}_{lj}^*) = \frac{\sum_{i=1}^{N} I(X_{ij} = h) P(\boldsymbol{\alpha}_{lj}^*|\mathbf{X}_i)}{\sum_{i=1}^{N} P(\boldsymbol{\alpha}_{lj}^*|\mathbf{X}_i)}$$

the marginal likelihood estimates of $S_j(h|\boldsymbol{\alpha}_{lj}^*)$

least-squares method

item parameter φ

expected a posteriori (EAP)

individuals' attribute patterns

Relations with existing polytomous CDMs

- US-GDINA

  - NRDM (the processing function is the G-DINA model)

  - PC-DINA model (the processing function is the DINA model)

$$P(X_j = h|\boldsymbol{\alpha}_{lj}^*) = \delta_{jh0} + \sum_{k=1}^{K_j^*} \delta_{jhk}\alpha_{lk} + \sum_{k'=k+1}^{K_j^*} \sum_{k=1}^{K_j^*-1} \delta_{jhkk'}\alpha_{lk}\alpha_{lk'} + \ldots + \delta_{jh12\ldots K_j^*} \prod_{k=1}^{K_j^*} \alpha_{lk}$$

Study 1 - purpose

Parameters of The Sequential G-DINA

Whether parameters of the sequential G-DINA model can be recovered accurately based on the proposed estimation algorithm?

Person Classifications

Whether the sequential G-DINA model can provide more accurate person classifications than the G-DINA model using dichotomized responses?

Parameter Recovery

Whether the attribute and category association can be used to improve parameter recovery for the sequential G-DINA model?

Study 1 - design

- **Independent variables**
  - **Sample size** (N = 500, 1,000, 2,000 or 4,000)

    ➡ uniform attribute distribution & RS-GDINA model
  - **Item quality** (high, moderate, low)

$$S_j(h|\boldsymbol{\alpha}_{ljh}^* = \mathbf{1}) = .9 \text{ and } S_j(h|\boldsymbol{\alpha}_{ljh}^* = \mathbf{0}) = .1$$
$$S_j(h|\boldsymbol{\alpha}_{ljh}^* = \mathbf{1}) = .8 \text{ and } S_j(h|\boldsymbol{\alpha}_{ljh}^* = \mathbf{0}) = .2$$
$$S_j(h|\boldsymbol{\alpha}_{ljh}^* = \mathbf{1}) = .7 \text{ and } S_j(h|\boldsymbol{\alpha}_{ljh}^* = \mathbf{0}) = .3$$

Study 1 - design

Table 3. Restricted Q_C-matrix for data simulation

Item	Category	A1	A2	A3	A4	A5	Item	Category	A1	A2	A3	A4	A5
1	1	1	0	0	0	0	11	1	1	1	0	0	0
1	2	0	1	0	0	0	11	2	0	0	0	0	1
2	1	0	0	1	0	0	12	1	1	1	1	0	0
2	2	0	0	0	1	0	12	2	0	0	0	1	1
3	1	0	0	0	0	1	13	1	1	1	0	0	0
3	2	1	0	0	0	0	13	2	0	0	1	1	1
4	1	0	0	0	0	1	14	1	1	0	1	0	0
4	2	0	0	0	1	0	14	2	0	0	0	1	0
5	1	0	0	1	0	0	14	3	0	0	0	0	1
5	2	0	1	0	0	0	15	1	0	0	0	0	1
6	1	1	0	0	0	0	15	2	0	0	1	1	0
6	2	0	1	1	0	0	15	3	0	1	0	0	0
7	1	0	0	1	0	0	16	1	1	0	0	0	0
7	2	0	0	0	1	1	16	2	0	1	0	0	0
8	1	0	0	0	0	1	16	3	0	0	1	1	0
8	2	1	1	0	0	0	17	1	1	0	0	0	0
9	1	0	0	0	1	1	18	1	0	1	0	0	0
9	2	0	0	1	0	0	19	1	0	0	1	0	0
10	1	0	1	0	1	0	20	1	0	0	0	1	0
10	2	1	0	0	0	0	21	1	0	0	0	0	1

Study 1 - design

- **Independent variables**
  - **Sample size** (N = 500, 1,000, 2,000 or 4,000)

    ➡ uniform attribute distribution & RS-GDINA model

  - **Item quality** (high, moderate, low)

$$S_j(h|\boldsymbol{\alpha}^*_{ljh} = \mathbf{1}) = .9 \text{ and } S_j(h|\boldsymbol{\alpha}^*_{ljh} = \mathbf{0}) = .1$$

$$S_j(h|\boldsymbol{\alpha}^*_{ljh} = \mathbf{1}) = .8 \quad \text{and} \quad S_j(h|\boldsymbol{\alpha}^*_{ljh} = \mathbf{0}) = .2$$

$$S_j(h|\boldsymbol{\alpha}^*_{ljh} = \mathbf{1}) = .7 \text{ and } S_j(h|\boldsymbol{\alpha}^*_{ljh} = \mathbf{0}) = .3$$

➡ monotonicity constraint

$$\boldsymbol{P}_{lj} = [P(X_j = 0|\boldsymbol{\alpha}^*_{lj}), \ldots, P(X_j = H_j|\boldsymbol{\alpha}^*_{lj})]$$

Dichotomously: Bernoulli
Polytomously: generalized Bernoulli

Study 1 - design

- **To fit the US-GDINA model**

Table 3. Restricted Q_C-matrix for data simulation

Item	Category	A1	A2	A3	A4	A5	Item	Category	A1	A2	A3	A4	A5
1	1	1	0	0	0	0	11	1	1	1	0	0	0
1	2	0	1	0	0	0	11	2	0	0	0	0	1
2	1	0	0	1	0	0	12	1	1	1	1	0	0
2	2	0	0	0	1	0	12	2	0	0	0	1	1
3	1	0	0	0	0	1	13	1	1	1	0	0	0
3	2	1	0	0	0	0	13	2	0	0	1	1	1
4	1	0	0	0	0	1	14	1	1	0	1	0	0
4	2	0	0	0	1	0	14	2	0	0	0	1	0
5	1	0	0	1	0	0	14	3	0	0	0	0	1
5	2	0	1	0	0	0	15	1	0	0	0	0	1
6	1	1	0	0	0	0	15	2	0	0	1	1	0
6	2	0	1	1	0	0	15	3	0	1	0	0	0
7	1	0	0	1	0	0	16	1	1	0	0	0	0
7	2	0	0	0	1	1	16	2	0	1	0	0	0

Study 1 - design

- **To fit the G-DINA model**

  - for one, partial credit and full marks were converted to 1;

  - for the other, only full marks were converted to 1, and partial credit was transformed to 0.

Study 1 - design

- **Dependent variables**
  - **RMSE** (the root mean square error)
  - only calculated for the sequential G-DINA model;

$$\text{RMSE} = \sqrt{\frac{\sum_{r=1}^{R}\sum_{c=1}^{2^K}\sum_{j=1}^{J}\left[\hat{P}^{(r)}(X_j = h|\boldsymbol{\alpha}_c) - P^{(r)}(X_j = h|\boldsymbol{\alpha}_c)\right]^2}{J \times 2^K \times R}},$$

  - **PCV** (the proportion of correctly classified attribute vectors)

$$\text{PCV} = \frac{\sum_{r=1}^{R}\sum_{i=1}^{N} I^{(r)}[\boldsymbol{\alpha}_i = \hat{\boldsymbol{\alpha}}_i]}{N \times R}$$

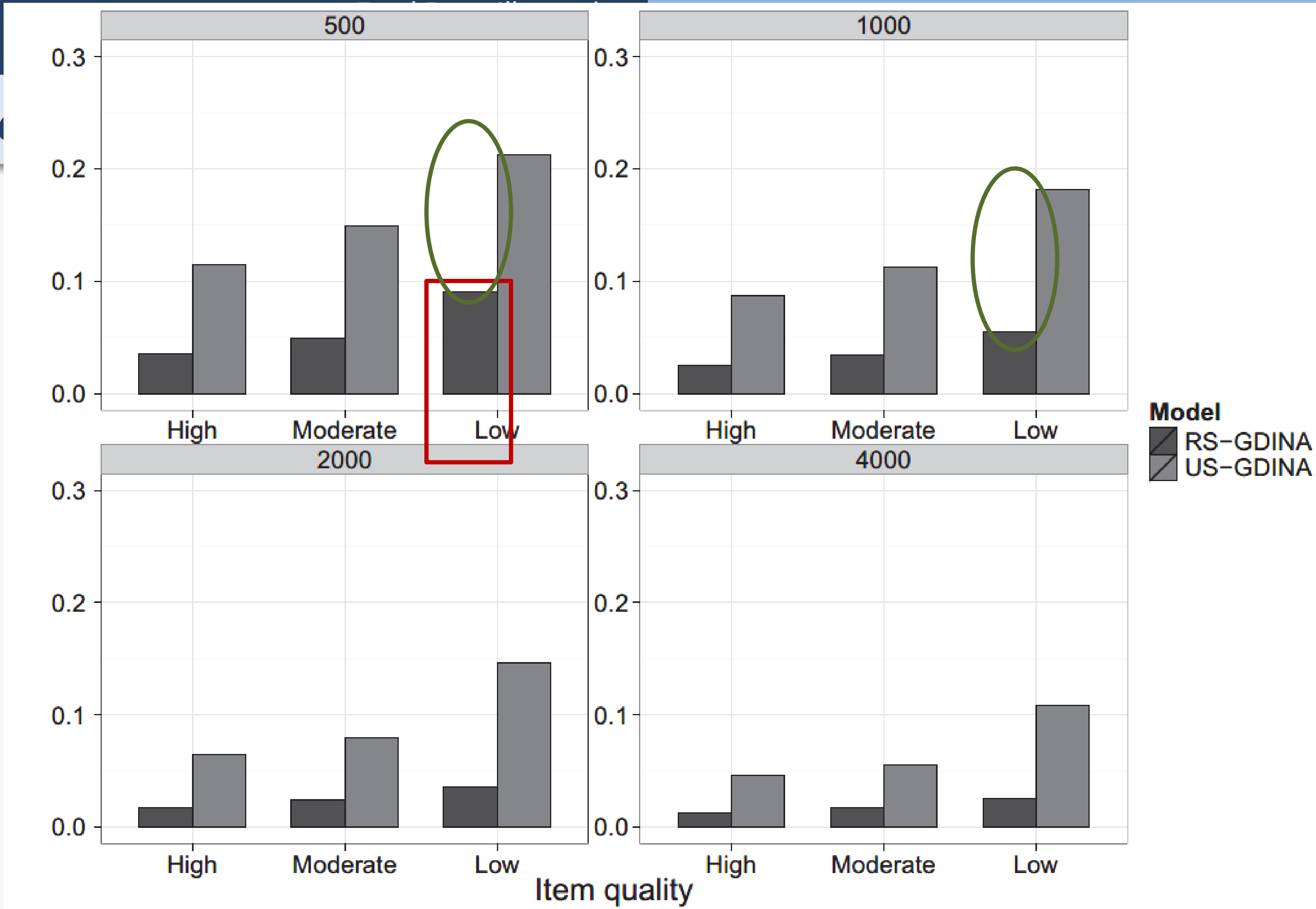


Figure 1. RMSE of the sequential G-DINA models.

Study 1 - results

Table 4. PCVs for the sequential G-DINA models and the G-DINA mode

partial full

N	Item quality	RS-GDINA	US-GDINA	GDINA1	GDINA2
500	High	.917	.901	.759	.742
	Moderate	.679	.601	.442	.371
	Low	.310	.223	.182	.142
1,000	High	.921	.911	.790	.752
	Moderate	.692	.641	.476	.415
	Low	.354	.254	.199	.147
2,000	High	.923	.918	.799	.761
	Moderate	.697	.674	.501	.441
	Low	.372	.295	.217	.152
4,000	High	.924	.921	.809	.765
	Moderate	.702	.690	.534	.456
	Low	.384	.339	.240	.168

Study 2 - purpose

The Impact of The Discrepancy

the impact of the discrepancy between the observed and predicted processing functions on parameter estimation

LRT, AIC, BIC

whether these indices can be used to select the appropriate sequential G-DINA model under various degrees of discrepancy

Study 2 - design

- **Independent variables**
  - **Sample size** (N = 500, 1,000, 2,000 or 4,000)
  - **Item quality** (high, moderate, low)
  - **magnitude of disturbances** (small, large)
  - the uncertainty in attribute and category association

    $\varepsilon \sim U[-0.1, 0.1] \rightarrow$ small disturbance; $\varepsilon = 0.1$

    $\varepsilon \sim U[-0.2, 0.2] \rightarrow$ large disturbance; $\varepsilon = 0.2$

Study 2 - design

- **Dependent variables**
  - **LRT** (0.05 significant level)
  - **AIC**
  - **BIC**

→

The proportion choosing the US-GDINA model for each statistic

The PCV based on the models selected by the LRT, AIC, and BIC

Study 2 - results

Table 5. Proportion choosing the US-GDINA model

		ε = 0			ε = 0.1			ε = 0.2		
N	Item quality	LRT	AIC	BIC	LRT	AIC	BIC	LRT	AIC	BIC
500	High	.14	.00	.00	1.00	.01	.00	1.00	1.00	.00
	Moderate	.90	.00	.00	1.00	.00	.00	1.00	.99	.00
	Low	1.00	.00	.00	1.00	.00	.00	1.00	.31	.00
1,000	High	.08	.00	.00	1.00	1.00	.00	1.00	1.00	.00
	Moderate	.57	.00	.00	1.00	.02	.00	1.00	1.00	.00
	Low	1.00	.00	.00	1.00	.01	.00	1.00	.98	.00
2,000	High	.02	.00	.00	1.00	1.00	.00	1.00	1.00	.98
	Moderate	.24	.00	.00	1.00	.90	.00	1.00	1.00	.14
	Low	1.00	.00	.00	1.00	.03	.00	1.00	1.00	.00
4,000	High	.06	.00	.00	1.00	1.00	.12	1.00	1.00	1.00
	Moderate	.11	.00	.00	1.00	1.00	.00	1.00	1.00	1.00
	Low	.96	.00	.00	1.00	.67	.00	1.00	1.00	.00

Study 2 - results

Table 5. Proportion choosing the US-GDINA model

		ε = 0			ε = 0.1			ε = 0.2		
N	Item quality	LRT	AIC	BIC	LRT	AIC	BIC	LRT	AIC	BIC
500	High	.14	.00	.00	1.00	.01	.00	1.00	1.00	.00
	Moderate	.90	.00	.00	1.00	.00	.00	1.00	.99	.00
	Low	1.00	.00	.00	1.00	.00	.00	1.00	.31	.00
1,000	High	.08	.00	.00	1.00	1.00	.00	1.00	1.00	.00
	Moderate	.57	.00	.00	1.00	.02	.00	1.00	1.00	.00
	Low	1.00	.00	.00	1.00	.01	.00	1.00	.98	.00
2,000	High	.02	.00	.00	1.00	1.00	.00	1.00	1.00	.98
	Moderate	.24	.00	.00	1.00	.90	.00	1.00	1.00	.14
	Low	1.00	.00	.00	1.00	.03	.00	1.00	1.00	.00
4,000	High	.06	.00	.00	1.00	1.00	.12	1.00	1.00	1.00
	Moderate	.11	.00	.00	1.00	1.00	.00	1.00	1.00	1.00
	Low	.96	.00	.00	1.00	.67	.00	1.00	1.00	.00

Study 2 - results

Table 6. PCVs of the sequential G-DINA models and selected models using the LRT, AIC, and BIC

		ε = 0.1					ε = 0.2				
N	Item quality	LRT	AIC	BIC	RS-GDINA	US-GDINA	LRT	AIC	BIC	RS-GDINA	US-GDINA
500	High	.927	.926	.926	.926	.927	.935	.935	.910	.910	.935
	Moderate	.644	.690	.690	.690	.644	.757	.757	.719	.719	.757
	Low	.252	.324	.324	.324	.252	.330	.347	.347	.347	.330
1,000	High	.934	.934	.928	.928	.934	.947	.947	.921	.921	.947
	Moderate	.676	.698	.698	.698	.676	.790	.790	.726	.726	.790
	Low	.286	.359	.360	.360	.286	.403	.403	.387	.387	.403
2,000	High	.940	.940	.930	.930	.940	.951	.951	.950	.923	.951
	Moderate	.709	.709	.707	.707	.709	.806	.806	.744	.733	.806
	Low	.335	.383	.384	.384	.335	.468	.468	.412	.412	.468
4,000	High	.943	.943	.934	.933	.943	.952	.952	.952	.925	.952
	Moderate	.720	.720	.708	.708	.720	.814	.814	.814	.734	.814
	Low	.384	.389	.396	.396	.384	.501	.501	.416	.416	.501

- Using **the LRT** can be lower than the upper benchmark by up to 10%;
- For **the BIC**, the maximum difference in PCV between the selected models and the upper benchmark is 8.5%;
- **The AIC** yielded the same PCV as the upper benchmark

Data

- booklets 4 and 5 of the Trends in International Mathematics and Science Study (TIMSS) 2007 fourth-grade mathematics assessment

- 823 students to 12 of 25 items involving eight of the original 15 attributes identified by Lee et al. (2011)

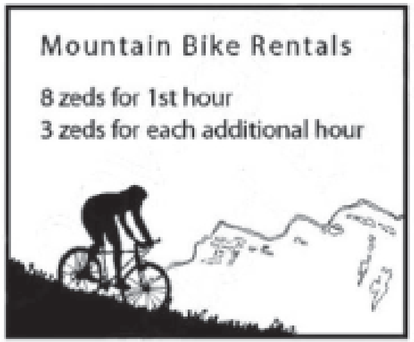
Data

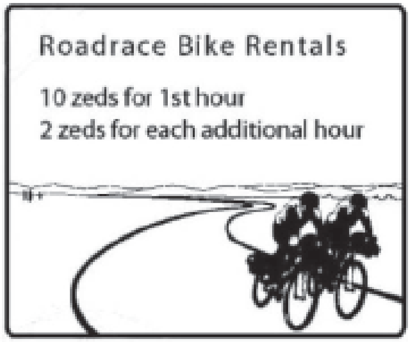
Table 7. Attribute definitions for TIMSS 2007 data

A1	Representing, comparing, and ordering whole numbers as well as demonstrating knowledge of place value
A2	Recognizing multiples, computing with whole numbers using the four operations, and estimating computations
A3	Solving problems, including those set in real-life contexts (e.g., measurement and money problems)
A4	Finding the missing number or operation and modelling simple situations involving unknowns in number sentence or expression
A5	Describing relationships in patterns and their extensions; generating pairs of whole numbers by a given rule and identifying a rule for every relationship given pairs of whole numbers
A6	Reading data from tables, pictographs, bar graphs, and pie charts
A7	Comparing and understanding how to use information from data.
A8	Understanding different representations and organizing data using tables, pictographs, and bar graphs

Source: Modified from Lee *et al.* (2011).

Data





A. Use the information in the posters to complete the tables.

Mountain Bike rental	
Hours	Cost (zeds)
1	8
2	11
3	
4	
5	
6	

Roadrace Bike Rentals	
Hours	Cost (zeds)
1	10
2	12
3	
4	
5	
6	

B. For what number of hours are the rental costs the same at the two clubs?

Answer: ____________________

- consider the two items as a single polytomous item to handle the testlet effect
- allow for answering item 7a successfully as a prerequisite to answering item 7b correctly for most

→ 820 students to 11 items were analysed

Method

Table 8. Q_C-matrix for TIMSS 2007 data

			Attributes							
Item	TIMSS item no.	Category	A1	A2	A3	A4	A5	A6	A7	A8
1	M041052	1	1	1	0	0	0	0	0	0
2	M041281	1	0	1	1	0	1	0	0	0
3a	**M041275**	**1**	**1**	**0**	**0**	**0**	**0**	**1**	**0**	**1**
3b	**M041275**	**2**	**1**	**0**	**0**	**0**	**0**	**1**	**0**	**1**
4	M031303	1	0	1	1	0	0	0	0	0
5	M031309	1	0	1	1	0	0	0	0	0
6	M031245	1	0	1	0	1	0	0	0	0
7a	**M031242A**	**1**	**0**	**1**	**1**	**0**	**1**	**0**	**0**	**0**
7b	**M031242B**	**2**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**0**
8	M031242C	1	0	1	1	0	1	0	1	0
9a	**M031247**	**1**	**0**	**1**	**1**	**1**	**0**	**0**	**0**	**0**
9b	**M031247**	**2**	**0**	**1**	**1**	**1**	**0**	**0**	**0**	**0**
10	M031173	1	0	1	1	0	0	0	0	0
11	M031172	1	1	1	0	0	0	1	0	1

Notes. Polytomous items are shown in bold. This Q_C-matrix is modified from Lee *et al.* (2011).

Results

Table 9. Estimates of processing functions for TIMSS 2007 data analysis

		Attribute pattern															
		0	1														
		00	10	01	11												
		000	100	010	001	110	101	011	111								
Item	Category	0000	1000	0100	0010	0001	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
1	1	.511	.941	.511	.999												
2	1	.260	.908	.444	.487	.908	.908	.617	.999								
3a	1	.002	.732	.999	.999	.999	.999	.999	.999								
3b	2	.013	.013	.540	.535	.999	.999	.999	.999								
4	1	.452	.868	.781	.973												
5	1	.122	.865	.305	.961												
6	1	.001	.001	.914	.999												
7a	1	.088	.621	.427	.854	.770	.899	.891	.955								
7b	2	.001	.999														
8	1	.257	.257	.377	.646	.347	.377	.999	.347	.988	.457	.646	.999	.718	.999	.999	.999
9a	1	.072	.145	.452	.072	.646	.145	.470	.646								
9b	2	.001	.645	.532	.532	.715	.762	.532	.762								
10	1	.095	.700	.761	.999												
11	1	.355	.835	.835	.355	.355	.835	.835	.835	.835	.835	.415	.835	.835	.835	.999	.999

Results

Table 9. Estimates of processing functions for TIMSS 2007 data analysis

		Attribute pattern																	
		0	1																
		00	10	01	11														
		000	100	010	001	110	101	011	111										
Item	**Category**	0000	1000	0100	0010	0001	1100	1010	1001	0110	0101	0011	1110	1101	1011	0111	1111
1	1	.511	.941	.511	.999														
2	1	.260	.908	.444	.487	.908	.908	.617	.999										
3a	1	.002	.732	.999	.999	.999	.999	.999	.999										
3b	2	.013	.013	.540	.535	.999	.999	.999	.999										
4	1	.452	.868	.781	.973														
5	1	.122	.865	.305	.961														
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7a	1	.088	.621	.427	.854	.770	.899	.891	.955										
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9a	1	.072	.145	.452	.072	.646	.145	.470	.646										
9b	2	.001	.645	.532	.532	.715	.762	.532	.762										
10	1	.095	.700	.761	.999														
11	1	.355	.835	.835	.355	.355	.835	.835	.835	.835	.835	.415	.835	.835	.835	.999	.999

Discussion

- Developed a new polytomous CDM for graded responses, the sequential G-DINA model.
- The sequential G-DINA model is also suitable for unordered categorical responses when the unrestricted Qc-matrix is used.
- The simulation study shows that the proposed estimation algorithm can produce accurate item and person parameter recovery.

Thanks for listening!

Yingshi Huang

2019/07/15