JEM JOURNAL of EDUCATIONAL MEASUREMENT
NCME national council on measurement in education
Journal of Educational Measurement
Winter 2015, Vol. 52, No. 4, pp. 457–476

Attribute-Level and Pattern-Level Classification Consistency and Accuracy Indices for Cognitive Diagnostic Assessment

Wenyi Wang, Lihong Song, Ping Chen, Yaru Meng, Shuliang Ding

01 December 2015
IF: 0.938

Reporter: Yingshi Huang

Introduction

- Cognitive diagnostic assessment (CDA)

reliability of diagnostic scores?

↓ criterion-referenced tests

classification consistency and accuracy indices for CDA

Cui, Gierl, & Chang (2012): P_c & P_a

1. at the whole-pattern level
2. difficult to calculate

↓

find a new method to estimate these indices to **cater to any test**, not only at the pattern level but also **at the attribute level**

Examinee Classification Methods for CDA

- Criterion-referenced test: score/ability
  - Classification Consistency (CC)

		Version B	
		pass	fail
Version A	pass	p_{11}	p_{10}
	fail	p_{01}	p_{00}

  - Classification Accuracy (CA)

		observed	
		pass	fail
true	pass	p_{11}	p_{10}
true	fail	p_{01}	p_{00}

➡ compute the expected probability of scoring in each category C: $\hat{p}_{iC}$

Examinee Classification Methods for CDA

- Criterion-referenced test: score/ability
  - Classification Consistency (CC)

		Version B	
		pass	fail
Version A	pass	p_{11}	p_{10}
	fail	p_{01}	p_{00}

  - Classification Accuracy (CA)

		observed	
		pass	fail
true	pass	p_{11}	p_{10}
true	fail	p_{01}	p_{00}

➡ compute the expected probability of scoring in each category C: $\hat{p}_{iC}$

- Cognitive diagnostic assessment: pattern/attribute
  - Classification Consistency (CC)

		Version B	
		master	otherwise
Version A	master	p_{11}	p_{10}
	otherwise	p_{01}	p_{00}

  - Classification Accuracy (CA)

		observed	
		master	otherwise
true	master	p_{11}	p_{10}
true	otherwise	p_{01}	p_{00}

➡ compute the attribute mastery probability: $\hat{p}_{ic}^{(k)}$

Examinee Classification Methods for CDA

- **Objective:** the probability that an examinee has (not) mastered the attribute k

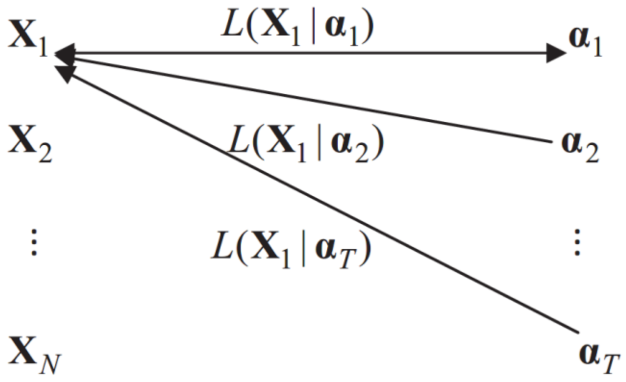
$$\hat{\mathbf{P}}^{(k)}_{N\times 2} = (\hat{p}^{(k)}_{ic}) = P(\alpha_c|\mathbf{X}_i)$$

- Under the assumption of local independence

$$L(\mathbf{X_i}|\alpha_\mathbf{i}) = P(\mathbf{X_i} = \mathbf{x}_i|\alpha_\mathbf{i}) = \prod_{j=1}^{M} P_j(\alpha_\mathbf{i})^{x_{ij}}(1 - P_j(\alpha_\mathbf{i}))^{1-x_{ij}}$$

$$P(\alpha_\mathbf{c}|\mathbf{X_i}) \propto L(\mathbf{X_i}|\alpha_\mathbf{c})p(\alpha_\mathbf{c})$$

$$P_j(\alpha_\mathbf{i}) = P(X_{ij} = 1|\alpha_\mathbf{i}, \mathbf{q}_j, \beta_j)$$



Examinee Classification Methods for CDA

- **Objective:** the probability that an examinee has (not) mastered the attribute k

$$\hat{\mathbf{P}}^{(k)}_{N\times 2} = (\hat{p}^{(k)}_{ic}) = P(\alpha_c|\mathbf{X}_i)$$

- Under the assumption of local independence

$$L(\mathbf{X_i}|\alpha_\mathbf{i}) = P(\mathbf{X_i} = \mathbf{x}_i|\alpha_\mathbf{i}) = \prod_{j=1}^{M} P_j(\alpha_\mathbf{i})^{x_{ij}}(1 - P_j(\alpha_\mathbf{i}))^{1-x_{ij}}$$

$$P(\alpha_\mathbf{c}|\mathbf{X_i}) \propto L(\mathbf{X_i}|\alpha_\mathbf{c})p(\alpha_\mathbf{c})$$

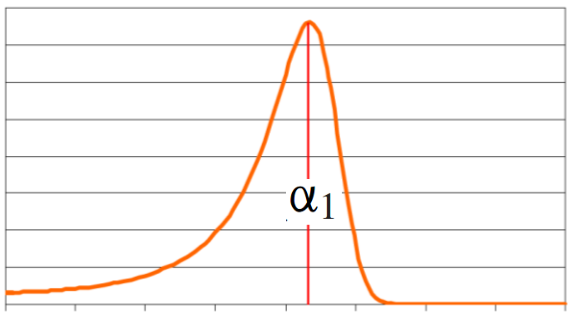
$$P_j(\alpha_\mathbf{i}) = P(X_{ij} = 1|\alpha_\mathbf{i}, \mathbf{q}_j, \beta_j)$$

  - maximum a posteriori (MAP)

$$\hat{\alpha}_\mathbf{i} = \underset{\alpha_\mathbf{c}\in\mathbf{Q_s}}{\arg\max}[P(\alpha_\mathbf{c}|\mathbf{X_i})]$$

  - marginal posterior probability (MPP)

$$\hat{p}_{ik} = \sum_{\alpha_c\in\mathbf{Q_s}} P(\alpha_c|\mathbf{X}_i)\mathbf{I}(\alpha_{ck} = 1)$$



[Guo, 2006 *Practical Assessment, Research and Evaluation*]

$$mp_k = \sum_{t=1}^{2^K} P(\alpha_{tk} = 1 | X_i)$$

$$\hat{\alpha}_{ik} = \begin{cases} 1, 如果 mp_k 大于等于 cutscore \\ 0, 如果 mp_k 小于 cutscore \end{cases} \quad k = 1, 2, \ldots, K$$

中国基础教育质量监测协同创新中心 陈 平

Attribute- and Pattern-Level CA

- to flag the status of the examinee on attribute

$$\mathbf{W} = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1C} \\ w_{21} & w_{22} & \cdots & w_{2C} \\ \vdots & \vdots & \cdots & \vdots \\ w_{N\,1} & w_{N\,2} & \cdots & w_{N\,C} \end{bmatrix}$$

- Attribute-Level

$$\hat{\mathbf{W}}^{(k)}_{N\times 2}: \ \hat{W}^{(k)}_i = (\mathbf{I}(\hat{\alpha}_{ik} = 0), \mathbf{I}(\hat{\alpha}_{ik} = 1))$$

$$\hat{\mathbf{P}}^{(k)}_{N\times 2}: \ \hat{\mathbf{P}}^{(k)}_i = (1 - \hat{p}_{ik}, \hat{p}_{ik})$$

$$\hat{\tau}_k = \frac{\sum_i \sum_c \left(\mathbf{P}^{(k)}_{N\times 2} .* \mathbf{W}^{(k)}_{N\times 2}\right)}{N}$$

- examinee (i = 1)

$$\hat{p}_{1k} = 0.85$$

$$\hat{W}^{(k)}_1 = (0, 1)$$

$$\hat{P}^{(k)}_1 = (1 - \hat{p}_{1k}, \hat{p}_{1k}) = (.15, .85)$$

- examinee (i = 2)

$$\hat{p}_{2k} = 0.2$$

$$\hat{W}^{(k)}_2 = (1, 0)$$

$$\hat{P}^{(k)}_2 = (1 - \hat{p}_{2k}, \hat{p}_{2k}) = (0.8, 0.2)$$

- $\hat{\tau}_k = (\hat{p}_{1k} + 1 - \hat{p}_{2k})/2 = .825$

Attribute- and Pattern-Level CA

- to flag the status of the examinee on attribute

$$\mathbf{W} = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1C} \\ w_{21} & w_{22} & \cdots & w_{2C} \\ \vdots & \vdots & \cdots & \vdots \\ w_{N\,1} & w_{N\,2} & \cdots & w_{N\,C} \end{bmatrix}$$

- MAP method

$$\hat{\tau}_i = \sum_c (\hat{p}_{ic} * \hat{w}_{ic}) = \max(\hat{\mathbf{P}}_i)$$

$$\hat{\tau} = \sum_i \hat{\tau}_i / N$$

- Attribute-Level

$$\hat{\mathbf{W}}^{(k)}_{N\times 2}: \; \hat{W}^{(k)}_i = (\mathbf{I}(\hat{\alpha}_{ik} = 0), \mathbf{I}(\hat{\alpha}_{ik} = 1))$$

$$\hat{\mathbf{P}}^{(k)}_{N\times 2}: \; \hat{\mathbf{P}}^{(k)}_i = (1 - \hat{p}_{ik}, \hat{p}_{ik})$$

$$\hat{\tau}_k = \frac{\sum_i \sum_c \left(\mathbf{P}^{(k)}_{N\times 2} .* \mathbf{W}^{(k)}_{N\times 2}\right)}{N}$$

- Pattern-Level

$$\hat{\mathbf{W}}_{N\times T}: \; \hat{W}_i = (\mathbf{I}(\hat{\alpha}_i = \alpha_c))$$

$$\hat{\mathbf{P}}_{N\times T}: \; \hat{\mathbf{P}}_i = (\hat{p}_{ic}) = (P(\alpha_{\mathbf{c}} | \mathbf{X_i}))$$

$$\hat{\tau} = \frac{\sum_i \sum_c \left(\hat{\mathbf{P}}_{N\times T} .* \hat{\mathbf{W}}_{N\times T}\right)}{N}$$

Attribute- and Pattern-Level CC

- Attribute-Level

$$\hat{\mathbf{P}}^{(k)}_{N\times 2}: \quad \hat{\mathbf{P}}^{(k)}_{i} = (1-\hat{p}_{ik}, \hat{p}_{ik})$$

$$\hat{\gamma}_k = \frac{\sum_i \sum_c \left(\mathbf{P}^{(k)}_{N\times 2} . * \mathbf{P}^{(k)}_{N\times 2}\right)}{N}$$

  - the marginal posterior probabilities of an attribute k being mastered on either test are **identical**

$$\hat{p}^{(1)}_{ik} = \hat{p}^{(2)}_{ik}$$

$$\hat{p}_{ik} = 0.8$$

$$\hat{\gamma}_{ik} = 0.2 \times 0.2 + 0.8 \times 0.8$$

- Pattern-Level

$$\hat{\mathbf{P}}_{N\times T}: \quad \hat{\mathbf{P}}_{i} = (\hat{p}_{ic}) = (P(\alpha_{\mathbf{c}}|\mathbf{X_i}))$$

$$\hat{\gamma} = \frac{\sum_i \sum_c (\hat{\mathbf{P}}_{N\times T} . * \hat{\mathbf{P}}_{N\times T})}{N}$$

		Version B	
		master	otherwise
Version A	master	p_{11}	p_{10}
	otherwise	p_{01}	p_{00}

Relationships Among the Variance of Error and Accuracy

- the variance of error

  - $\hat{\sigma}_{ek}^2 = \sum_{i=1}^N \hat{p}_{ik}(1-\hat{p}_{ik})/N$

$$= \boxed{\sum_{i=1}^N \hat{p}_{ik}/N} - \sum_{i=1}^N \hat{p}_{ik}^2/N = \hat{\tau}_k - \sum_{i=1}^N \hat{p}_{ik}^2/N$$

$$\boxed{\begin{aligned}\hat{\tau}_k &= \frac{\sum_i \sum_c \left(\mathbf{P}_{N\times 2}^{(k)} .* \mathbf{W}_{N\times 2}^{(k)}\right)}{N} \\ &= \frac{1}{N}\sum_{i=1}^{N} \hat{p}_{ik}\end{aligned}}$$

- the Cauchy–Schwarz inequality

$$\left(\sum_{i=1}^N (1/N)^2\right)\left(\sum_{i=1}^N \hat{p}_{ik}^2\right) \geq \left(\sum_{i=1}^N \frac{1}{N}\hat{p}_{ik}\right)^2$$

$$\sum_{i=1}^N \hat{p}_{ik}^2/N \geq \hat{\tau}_k^2$$

$$-\sum_{i=1}^N \hat{p}_{ik}^2/N \leq -\hat{\tau}_k^2$$

$$\hat{\tau}_k - \sum_{i=1}^N \hat{p}_{ik}^2/N \leq \hat{\tau}_k - \hat{\tau}_k^2$$

$$\sigma_{ek}^2 \leq \hat{\tau}_k(1-\hat{\tau}_k)$$

$$\Rightarrow \quad \frac{1-\sqrt{1-4\hat{\sigma}_{ek}^2}}{2} \leq \hat{\tau}_k \leq \frac{1+\sqrt{1-4\hat{\sigma}_{ek}^2}}{2}$$

An Alternative Approach to Constructing Attribute-Level Indices

Can Cui's pattern-level indices be generalized to the attribute level?

- Latent class C_h : is similar to an equivalent class of **AMPs** $\boxed{\alpha_c} \rightarrow (\alpha_1, \alpha_2, \ldots, \alpha_K)$

  $C_h : h = 1, 2, \ldots, H$

  - when the Q-matrix of the test is a complete (or necessary and sufficient) Q-matrix: $H = 2^K$
  - otherwise: $H < 2^K$
  - π_h : all possible item response patterns that would be classified into C_h

- we have known: $P(\mathbf{X} = \mathbf{x} | \alpha_c) = \prod_{j=1}^{J} [P_j(\alpha_c)]^{x_j} [Q_j(\alpha_c)]^{1-x_j}$

- we want to know: $P(\mathbf{X} \in \pi_h | \alpha_c) = \sum_{\mathbf{x} \in \pi_h} P(\mathbf{X} = \mathbf{x} | \alpha_c)$

An Alternative Approach to Constructing Attribute-Level Indices

- Classification consistency
  - when $\mathbf{X}_1$ and $\mathbf{X}_2$ belong to the same latent class

$$P(\mathbf{X}_1 \in \pi_h, \mathbf{X}_2 \in \pi_h | \alpha_c) = \left(\boxed{\sum_{\mathbf{x} \in \pi_h} P(\mathbf{X} = \mathbf{x} | \alpha_c)}\right)^2$$

collapsing all H latent classes

$$P_c(\alpha_c) = \sum_{h=1}^{H} \left(\sum_{\mathbf{x} \in \pi_h} P(\mathbf{X} = \mathbf{x} | \alpha_c) \right)^2$$

collapsing all AMPs

$$P_c = \sum_{\alpha_c \in \mathbf{Q_s}} \left[\sum_{h=1}^{H} \left(\sum_{\mathbf{x} \in \pi_h} P(\mathbf{X} = \mathbf{x} | \alpha_c) \right)^2 \right] \hat{r}_{\alpha_\mathbf{c}}$$

$$\hat{r}_{\alpha_\mathbf{c}} = \sum_i P(\alpha_\mathbf{c} | \mathbf{X_i}) / N$$

(the relative frequency of AMP)

- Classification accuracy
  - true latent class: C_t

$$P(\mathbf{X} \in \pi_t | \alpha_c) = \boxed{\sum_{\mathbf{x} \in \pi_t} P(\mathbf{X} = \mathbf{x} | \alpha_c)}$$

collapsing all AMPs

$$P_a = \sum_{\alpha_c \in Q_s} \left[\sum_{\mathbf{x} \in \pi_t} P(\mathbf{X} = \mathbf{x} | \alpha_c) \right] \hat{r}_{\alpha_\mathbf{c}}$$

require the summation
over 2^J item response patterns

An Alternative Approach to Constructing Attribute-Level Indices

- Pattern-Level: $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_K]$

$$\hat{r}_{k1} = \sum_{\alpha_c \in \mathbf{Q_s}} \left[\mathbf{I}(\alpha_{ck} = 1)\hat{r}_{\alpha_c}\right]$$

$$\hat{r}_{k0} = 1 - \hat{r}_{k1}$$

- Attribute-Level: $\alpha_k = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\begin{array}{cc} \alpha_2 & \\ [0, \mathbf{1}, 0, 0, 1] & \hat{r}_{\alpha_{c1}} \\ [0, \mathbf{1}, 0, 1, 0] & \hat{r}_{\alpha_{c2}} \\ \vdots & \vdots \\ [\alpha_1, \mathbf{1}, \alpha_3, \ldots, \alpha_k] & \hat{r}_{\alpha_{cT}} \end{array}$$

- Classification consistency

$$P_{ck1} = \frac{\sum_{\alpha_c \in \mathbf{Q_s}} \left[P_c(\alpha_c)\mathbf{I}(\alpha_{ck} = 1)\hat{r}_{\alpha_c}\right]}{\hat{r}_{k1}}$$

$$P_{ck0} = \frac{\sum_{\alpha_c \in \mathbf{Q_s}} \left[P_c(\alpha_c)\mathbf{I}(\alpha_{ck} = 0)\hat{r}_{\alpha_c}\right]}{\hat{r}_{k0}}$$

$$P_{ck} = (P_{ck1})^2\hat{r}_{k1} + (P_{ck0})^2\hat{r}_{k0}$$

- Classification accuracy

$$P_{ak1} = \frac{\sum_{\alpha_c \in \mathbf{Q_s}} \left[P(\mathbf{X} \in \pi_t | \alpha_c)\mathbf{I}(\alpha_{ck} = 1)\hat{r}_{\alpha_c}\right]}{\hat{r}_{k1}}$$

$$P_{ak0} = \frac{\sum_{\alpha_c \in \mathbf{Q_s}} \left[P(\mathbf{X} \in \pi_t | \alpha_c)\mathbf{I}(\alpha_{ck} = 0)\hat{r}_{\alpha_c}\right]}{\hat{r}_{k0}}$$

$$P_{ak} = P_{ak1}\hat{r}_{k1} + P_{ak0}\hat{r}_{k0}$$

Simulation Study

- Questions

1. How close does the classification accuracy **match with the correct** classification rate?

2. How close does the classification consistency **match with the test-retest** consistency rate?

3. Are the new indices sensitive to changes in **test discrimination power, test length, and so on**?

4. How do the new indices perform **compared with Cui's indices**?

Simulation Study

- Method
  - under the deterministic inputs, noisy "and" gate (DINA) model

- Simulation Design
  - **total number of attributes (3)**:

    3 with p=0.5; 5 with p=0.3; 8 with p=0.1825
  - **item discrimination power (2)**:

    high: g & s ~ U(0.05, 0.25)

    low: g & s ~ U(0.25, 0.45)
  - **test discrimination power (2)**:

    high: using the cognitive diagnostic index (CDI)

    low: using a random way (RD)
  - **dependency among the attributes (2)**:

    independent: 0

    correlated: 0.5
  - **test length (4)**:

    5 items, 10 items, 15 items, 20 items

Simulation Study

- Evaluation Criteria: correct classification rate & test-retest consistency rate
  - **Pattern correct classification rate (PCCR)**

$$\mathrm{PCCR} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{I}(\hat{\alpha}_i = \alpha_i)$$

  - **Attribute correct classification rate (ACCR)**

$$\mathrm{ACCR}_k = \frac{1}{N}\sum_{i=1}^{N}\mathbf{I}(\hat{\alpha}_{ik} = \alpha_{ik})$$

  - **Pattern test-retest consistency rate (PTRCR)**

$$\mathrm{PTRCR}_{1,2} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{I}(\hat{\alpha}_i^{(1)} = \hat{\alpha}_i^{(2)})$$

  - **Attribute test-retest consistency rate (ATRCR)**

$$\mathrm{ATRCR}_{k,1,2} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{I}(\hat{\alpha}_{ik}^{(1)} = \hat{\alpha}_{ik}^{(2)})$$

$$C_{200}^{2} = 200 \times (200 - 1)/2$$

Results

Table 1

Pattern-Level Classification Consistency and Accuracy Indices Under Various Conditions When the Number of Attributes Is Five

				Classification Accuracy			Classification Consistency		
ρ	Item	Test	Length	PCCR	P_a	τ	PTRCR	P_c	γ
0	High	RD	5	.21	.28	.27	.29	.29	.16
			10	.55	.54	.51	.34	.33	.32
			15	.65	.64	.62	.45	.45	.46
			20	.64	.70	.66	.46	.54	.54
		CDI	5	.13	.20	.20	.56	.67	.18
			10	.39	.46	.50	.67	.70	.45
			15	.78	.64	.75	.73	.75	.69
			20	.82	.79	.83	.80	.82	.77
	Low	RD	5	.11	.08	.08	.08	.09	.04
			10	.15	.16	.16	.06	.06	.07
			15	.19	.19	.19	.07	.08	.09
			20	.23	.22	.22	.09	.09	.12
		CDI	5	.12	.12	.12	.12	.18	.07
			10	.25	.24	.24	.10	.10	.13
			15	.31	.29	.30	.13	.14	.17
			20	.37	.36	.37	.17	.17	.22
.5	High	RD	5	.18	.14	.17	.37	.47	.13
			10	.41	.37	.38	.30	.31	.28
			15	.63	.52	.57	.45	.44	.47
			20	.76	.67	.73	.60	.53	.64
		CDI	5	.21	.20	.20	.56	.67	.18
			10	.50	.46	.51	.68	.70	.46
			15	.82	.64	.76	.72	.75	.70
			20	.88	.79	.82	.81	.82	.76
	Low	RD	5	.02	.06	.06	.12	.16	.04
			10	.14	.13	.15	.09	.09	.07
			15	.18	.18	.19	.09	.08	.10
			20	.21	.20	.21	.11	.09	.11
		CDI	5	.12	.12	.12	.12	.18	.07
			10	.24	.24	.24	.10	.10	.13
			15	.30	.29	.30	.14	.14	.18
			20	.37	.36	.37	.17	.17	.22

Results

Table 1

Pattern-Level Classification Consistency and Accuracy Indices Under Various Conditions When the Number of Attributes Is Five

				Classification Accuracy			Classification Consistency		
ρ	Item	Test	Length	PCCR	P_a	τ	PTRCR	P_c	γ
0	High	RD	5	.21	.28	.27	.29	.29	.16
			10	.55	.54	.51	.34	.33	.32
			15	.65	.64	.62	.45	.45	.46
			20	.64	.70	.66	.46	.54	.54
		CDI	5	.13	.20	.20	.56	.67	.18
			10	.39	.46	.50	.67	.70	.45
			15	.78	.64	.75	.73	.75	.69
			20	.82	.79	.83	.80	.82	.77
	Low	RD	5	.11	.08	.08	.08	.09	.04
			10	.15	.16	.16	.06	.06	.07
			15	.19	.19	.19	.07	.08	.09
			20	.23	.22	.22	.09	.09	.12
		CDI	5	.12	.12	.12	.12	.18	.07
			10	.25	.24	.24	.10	.10	.13
			15	.31	.29	.30	.13	.14	.17
			20	.37	.36	.37	.17	.17	.22

				Classification Accuracy			Classification Consistency		
ρ	Item	Test	Length	PCCR	P_a	τ	PTRCR	P_c	γ
.5	High	RD	5	.18	.14	.17	.37	.47	.13
			10	.41	.37	.38	.30	.31	.28
			15	.63	.52	.57	.45	.44	.47
			20	.76	.67	.73	.60	.53	.64
		CDI	5	.21	.20	.20	.56	.67	.18
			10	.50	.46	.51	.68	.70	.46
			15	.82	.64	.76	.72	.75	.70
			20	.88	.79	.82	.81	.82	.76
	Low	RD	5	.02	.06	.06	.12	.16	.04
			10	.14	.13	.15	.09	.09	.07
			15	.18	.18	.19	.09	.08	.10
			20	.21	.20	.21	.11	.09	.11
		CDI	5	.12	.12	.12	.12	.18	.07
			10	.24	.24	.24	.10	.10	.13
			15	.30	.29	.30	.14	.14	.18
			20	.37	.36	.37	.17	.17	.22

Results

- The average MADs across all attributes:

P_{ak}	$\hat{\tau}_k$	P_{ck}	$\hat{\gamma}_k$
.0239	.0236	.0225	.0619

Table 2

Attribute-Level Classification Consistency and Accuracy Indices for Attribute 1 Under Various Conditions When the Number of Attributes Is Five

				Classification Accuracy			Classification Consistency		
ρ	Item	Test	Length	ACCR	P_{ak}	τ_k	ATRCR	P_{ck}	γ_k
0	High	RD	5	.49	.66	.66	.63	.57	.56
			10	.89	.84	.83	.74	.73	.73
			15	.89	.89	.86	.80	.80	.80
			20	.88	.89	.87	.80	.80	.80
		CDI	5	.94	.94	.94	.88	.88	.93
			10	.94	.96	.96	.92	.92	.95
			15	.98	.99	.98	.97	.97	.97
			20	.99	.99	.98	.97	.97	.97
	Low	RD	5	.62	.58	.58	.57	.53	.52
			10	.73	.72	.72	.61	.60	.61
			15	.72	.74	.74	.64	.63	.64
			20	.73	.74	.74	.63	.62	.65
		CDI	5	.60	.68	.72	.57	.64	.66
			10	.78	.76	.78	.65	.64	.69
			15	.78	.76	.78	.65	.64	.69
			20	.81	.82	.81	.72	.70	.74
.5	High	RD	5	.66	.73	.74	.78	.63	.66
			10	.70	.75	.76	.75	.64	.68
			15	.89	.91	.90	.83	.84	.85
			20	.95	.95	.94	.91	.91	.91
		CDI	5	.94	.94	.94	.88	.88	.93
			10	.94	.96	.96	.93	.93	.95
			15	.99	.99	.98	.97	.97	.97
			20	.99	.99	.98	.97	.97	.97
	Low	RD	5	.44	.52	.52	.65	.64	.50
			10	.60	.62	.66	.59	.56	.58
			15	.72	.71	.71	.61	.59	.62
			20	.72	.71	.71	.62	.59	.62
		CDI	5	.59	.68	.72	.56	.64	.66
			10	.77	.76	.78	.65	.64	.69
			15	.77	.76	.78	.65	.64	.69
			20	.81	.82	.81	.71	.70	.74

Results

Table 3

Classification Accuracy and Consistency Indices With Three and Eight Attributes When Test Length is 20

				Pattern Indices						Mean of Attribute Indices					
				Accuracy			Consistency			Accuracy			Consistency		
K	ρ	Item	Test	PCCR	P_a	τ	PTRCR	P_c	γ	ACCR	P_{ak}	τ_k	ATRCR	P_{ck}	γ_k
3	0	High	RD	.89	.88	.86	.81	.80	.79	.96	.96	.95	.93	.92	.92
			CDI	.99	.99	.99	.99	.98	.99	1.00	1.00	1.00	1.00	1.00	1.00
		Low	RD	.54	.49	.51	.33	.36	.38	.79	.78	.78	.69	.67	.70
			CDI	.67	.65	.67	.48	.48	.58	.87	.86	.87	.77	.77	.82
	.5	High	RD	.63	.79	.72	.87	.80	.70	.89	.88	.90	.90	.82	.88
			CDI	.99	.99	.99	.99	.98	.99	1.00	1.00	1.00	1.00	1.00	1.00
		Low	RD	.50	.49	.50	.32	.31	.37	.78	.77	.78	.67	.65	.70
			CDI	.65	.65	.67	.49	.48	.58	.87	.86	.87	.77	.77	.82
8	0	High	RD	.31	.34	.33	.15	.18	.20	.86	.86	.87	.78	.77	.81
			CDI	.45	.49	.52	.48	.51	.43	.91	.92	.92	.92	.88	.90
		Low	RD	.06	.05	.05	.01	.02	.02	.68	.66	.67	.59	.57	.58
			CDI	.11	.07	.08	.03	.07	.03	.73	.72	.73	.70	.70	.65
	.5	High	RD	.44	.36	.38	.22	.24	.28	.89	.87	.87	.82	.79	.83
			CDI	.48	.49	.52	.47	.51	.44	.91	.92	.92	.92	.88	.90
		Low	RD	.05	.03	.04	.02	.02	.01	.62	.60	.62	.58	.56	.55
			CDI	.10	.07	.08	.03	.07	.03	.74	.72	.73	.70	.70	.65

Discussion

- Provides useful estimates of CC and CA indices not only at the pattern level but also **at the attribute level**.
- The values of the new indices are **easier to calculate**.

Further Research

- solve a **practical problem** in test development
- reexamined in a **new context** or with **different groups**
- construct their **confidence intervals**
- be applied to **different CDMs**

Thanks for listening!