

A Continuous α -Stratification Index for Item Exposure Control in Computerized Adaptive Testing

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IF: 1.326

Reporter: Yingshi Huang

Applied Psychological Measurement

2018, Vol. 42(7) 523–537

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DOI: 10.1177/0146621618758289

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Alan Huebner

- Computerized Adaptive Testing (CAT)

practical consideration: prevent items from being overexposed

➡ *a*-stratification method: divide the item bank into K strata

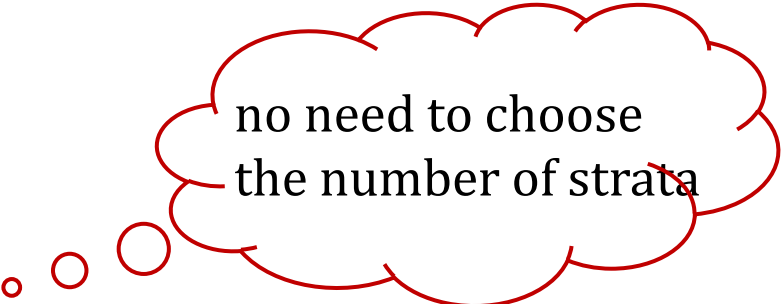
an appropriate K value ?

- balance exposure control and measurement accuracy
- moderate-size fixed constant ($K = 4$)

the replenishment of item bank ?

- recreate every time
- it is more challenging in high-stakes tests

propose an alternate approach: continuous *a*-stratification



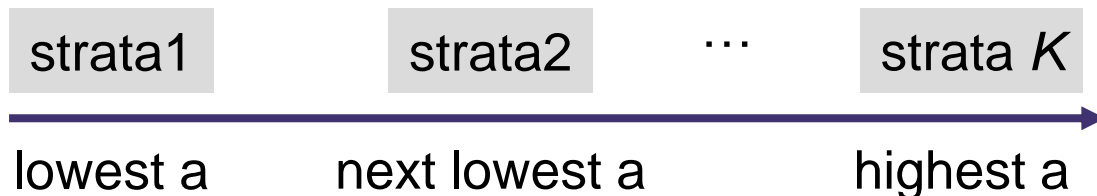
no need to choose
the number of strata

- One-dimensional IRT model

$$P_j(\theta) = P_j(U = 1 | \theta) = c_j + (1 - c_j) \frac{\exp[1.7a_j(\theta - b_j)]}{1 + \exp[1.7a_j(\theta - b_j)]}$$

- Method proceeds

1. Partition the item bank into K levels according to item a values;
2. Partition the test into K stages;
3. J_k items are administered from Stratum k ($J_1 + J_2 + \dots + J_k = J$): e.g. match- b / MFI;
4. Repeat Step 3 from $k = 1, 2, \dots, K$.



- Two-dimensional IRT model

$$P_j(\boldsymbol{\theta}) = P_j(U = 1 | \boldsymbol{\theta}) = \frac{\exp\left[1.7\mathbf{a}'_j(\boldsymbol{\theta} - b_j\mathbf{1})\right]}{1 + \exp\left[1.7\mathbf{a}'_j(\boldsymbol{\theta} - b_j\mathbf{1})\right]} \quad \mathbf{a}_j = \{a_{j1}, a_{j2}\} \quad \boldsymbol{\theta} = \{\theta_1, \theta_2\}$$



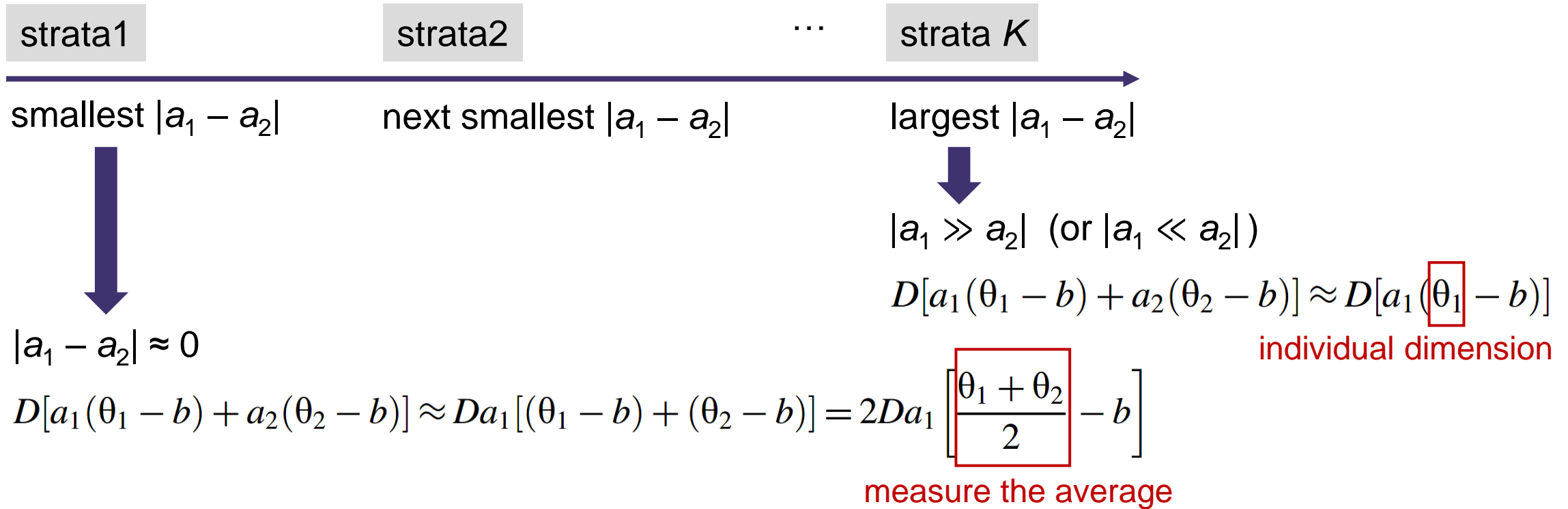
use some functions of \mathbf{a}_j for the stratification

What we want to achieve:

- At an early stage: **measure the average** of θ
- As the test progresses: pinpoint the location of θ along **each individual dimension**
- θ_1 and θ_2 should be measurable to the **same degree of precision**

- Method proceeds

1. Partition the item bank into K levels according to item a values;



- **mimics** the behavior of $|a|$ (equivalently a , because $a > 0$) in a **unidimensional test**

- Method proceeds

1. Partition the item bank into K levels according to item a values;
2. Divide each level (except the first stratum) into two subsections:

sub-1($a_1 > a_2$) & **sub-2**($a_1 \leq a_2$);

3. Partition the test into K stages;

4. J_k items are administered from Stratum k ($J_1 + J_2 + \dots + J_k = J$):

in the first stage: using some psychometric criterion

after the first stage: select J_{k1} and J_{k2} items from each subsection ($J_{k1} + J_{k2} = J_k$);

5. Repeat Step 4 from $k = 1, 2, \dots, K$. And keep $\sum_{k=2}^K J_{k1} = \sum_{k=2}^K J_{k2}$ → same degree of precision

– “match- b ”: $\hat{b} = \frac{a_1 \hat{\theta}_1 + a_2 \hat{\theta}_2}{a_1 + a_2}$

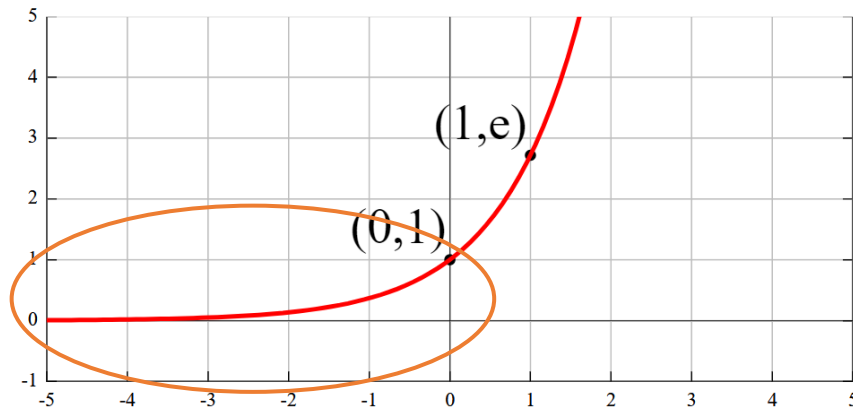
– D-optimality: $(1.7)^4 \left\{ \sum_{j=1}^{j'+1} a_{j1}^2 P_j(\hat{\theta}) (1 - P_j(\hat{\theta})) \times \sum_{j=1}^{j'+1} a_{j2}^2 P_j(\hat{\theta}) (1 - P_j(\hat{\theta})) - \left(\sum_{j=1}^{j'+1} a_{j1} a_{j2} P_j(\hat{\theta}) (1 - P_j(\hat{\theta})) \right)^2 \right\}$

- Continuous a-stratification index (CAI)
 - incorporate exposure control as one building block intrinsic to the index itself
 - item $(j' + 1)$ is selected to **maximize** the quantity:

CAI \times Inf

- For the 1-DIM case

$$\text{CAI} = \exp \left[-\beta \left(\frac{r(a)}{j'/J} - 1 \right)^2 \right] \quad \beta > 0: \text{ determines the sensitivity of the discrepancy between } j'/J \text{ and } a$$



➔ $\frac{r(a)}{j'/J} - 1$: find $r(a)$ as close as possible to j'/J

- ✓ At the beginning of the test:
 j'/J is small \rightarrow item with smaller a
- ✓ As the test proceeds:
force a to be ascending

- Continuous a-stratification index (CAI)

- For the 2-DIM case

$$\text{CAI} = \exp \left[-\beta \left(\frac{r(\mathbf{a}'\mathbf{a})}{j'/J} - 1 \right)^2 \right] \quad \sqrt{\mathbf{a}'\mathbf{a}} \text{ is the so-called multidimensional } a \text{ parameter}$$

- a second version

$$\left| \frac{r(a)}{j'/J} - 1 \right| \quad \text{vs} \quad \left(\frac{r(a)}{j'/J} - 1 \right)^2$$



Preliminary simulations: did not outperform in any of the test conditions

- Item selection method
 - CAI method ($\beta = 2$)
 - a-stratification method ($K = 4$):
Maximizing Fisher information (1-DIM) or D-optimality (2-DIM)
Match-b
 - $\hat{\theta}_0$: selecting randomly within the range -3.5 to 3.5

Case	Length	Test stages for Strata 1/2/3/4
1-DIM	20	(1-5)/(6-10)/(11-15)/(16-20)
	30	(1-7)/(8-14)/(15-22)/(23-30)
2-DIM	40	(1-10)/(11-20)/(21-30)/(31-40)
	60	(1-15)/(16-30)/(31-45)/(46-60)

Note. 1-DIM = one-dimensional; 2-DIM = two-dimensional.

- 1-DIM Study

- Item pool structure

- test length: 20 & 30 items
 - item bank: 500 items
 - $a \sim U(0.0, 1.3)$
 - $b \sim U(-1.3, 1.3)$
 - $c \sim U(0.2, 0.3)$

- Examinee generation

- examinee abilities:
 1. discrete uniform (DU) distribution:
 - from -3.2 to 3.2 by an increment of 0.4 ($17 \theta \times 300 = 5,100$)
 2. normal(0,1) distribution: 5,100 examinees
 - theta estimation: grid search (from -3.5 to 3.5, by 0.01)

$2 \times 2 \times 3 = 12$ total conditions

- Evaluation criteria

- Bias = $\frac{1}{N} \sum_{i=1}^N (\hat{\theta}_i - \theta_i)$
 - MSE = $\frac{1}{N} \sum_{i=1}^N (\hat{\theta}_i - \theta_i)^2$
 - $\chi^2 = \sum_{m=1}^M \frac{(er_m - J/M)^2}{J/M}$

- 2-DIM Study

- Item pool structure

- test length: 40 & 60 items
 - item bank: 500 items
 - a_1 & $a_2 \sim U(0.0, 1.3)$
 - $b \sim U(-1.3, 1.3)$

- Examinee generation

- examinee abilities:

- 1. 2-DIM grid:

- from -2 to 2 by an increment of 0.4 ($11 \theta_1 \times 11 \theta_2 \times 50 = 6,050$)

- 2. bivariate normal distribution: 6,050 examinees

- $\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right)$ low: $\rho = 0.3$; high: $\rho = 0.7$

- theta estimation: grid search (from -3.5 to 3.5, by 0.1)

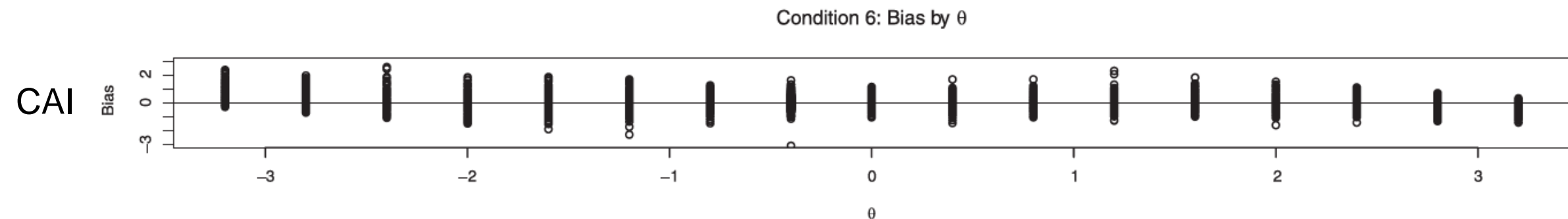
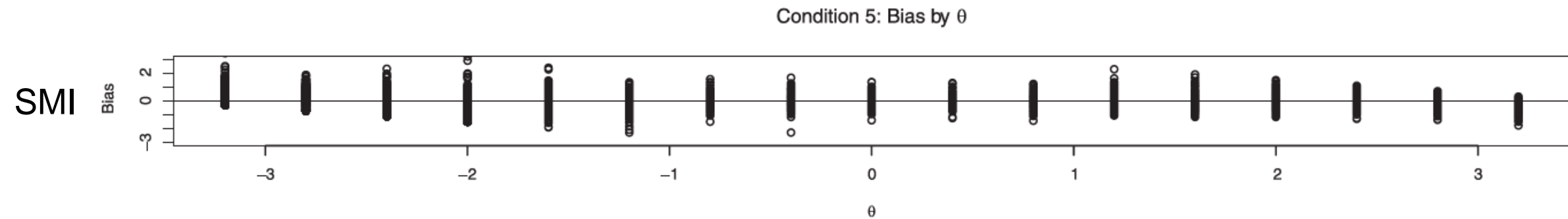
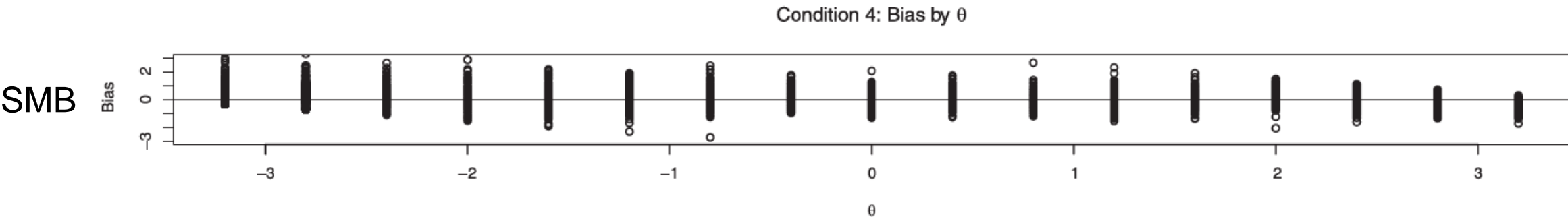
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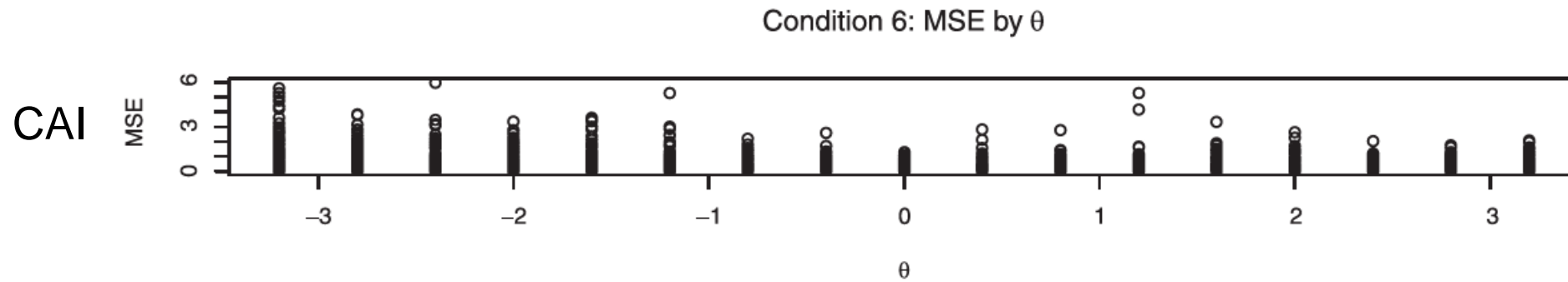
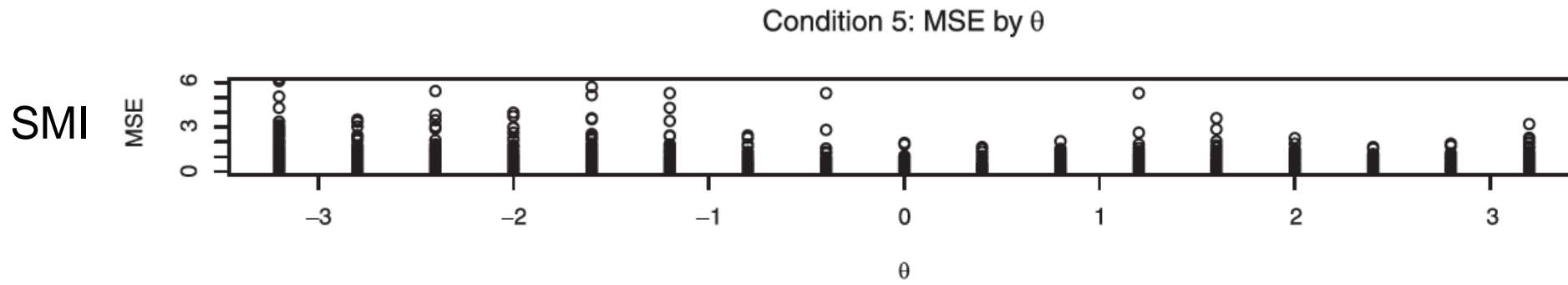
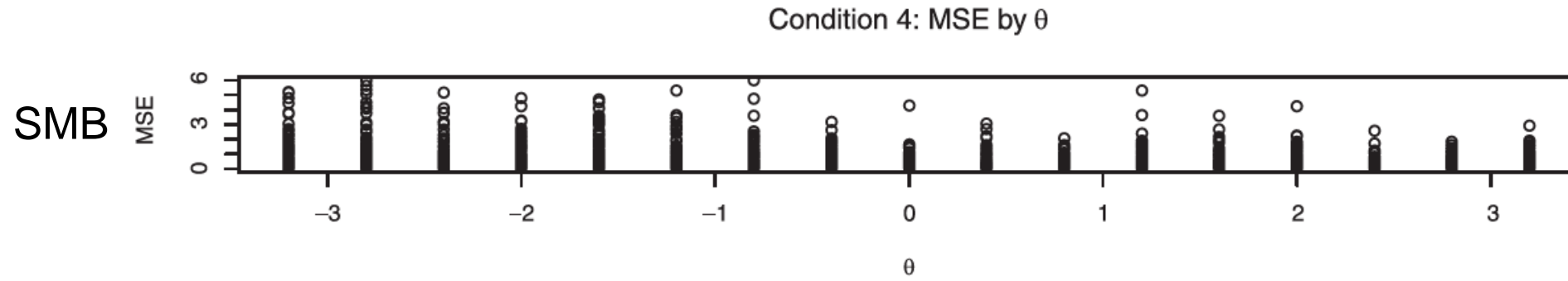
- Bias = $\frac{1}{N} \sum_{i=1}^N \{ (\hat{\theta}_{i1} - \theta_{i1}) + (\hat{\theta}_{i2} - \theta_{i2}) \}$
 - MSE = $\frac{1}{N} \sum_{i=1}^N \{ (\hat{\theta}_{i1} - \theta_{i1})^2 + (\hat{\theta}_{i2} - \theta_{i2})^2 \}$
 - $\chi^2 = \sum_{m=1}^M \frac{(er_m - J/M)^2}{J/M}$

2×3×3 = 18 total conditions

Condition	Length	Ability	Method	Bias	MSE	χ^2
1	20	$N(0,1)$	SMB	0.076	0.31	83.37
2	20	$N(0,1)$	SMI	0.032	0.22	145.03
3	20	$N(0,1)$	CAI	0.037	0.21	101.01
4	20	DU	SMB	0.085	0.44	115.17
5	20	DU	SMI	0.039	0.33	127.85
6	20	DU	CAI	0.040	0.32	103.47
7	30	$N(0,1)$	SMB	0.042	0.19	69.8
8	30	$N(0,1)$	SMI	0.021	0.14	139.57
9	30	$N(0,1)$	CAI	0.019	0.13	98.54
10	30	DU	SMB	0.057	0.28	105.65
11	30	DU	SMI	0.026	0.23	122.37
12	30	DU	CAI	0.031	0.24	99.48

Note. 1-DIM = one-dimensional; MSE = mean squared error; DU = discrete uniform distribution; SMB = α -stratification with match- b item selection; SMI = α -stratification with maximum Fisher information item selection; CAI = continuous α -stratification.



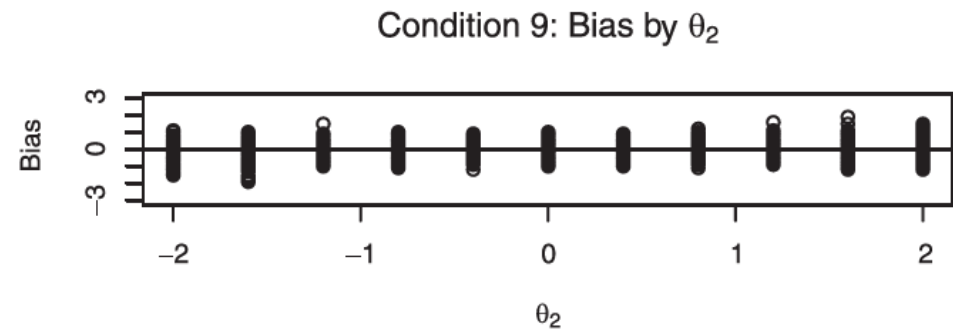
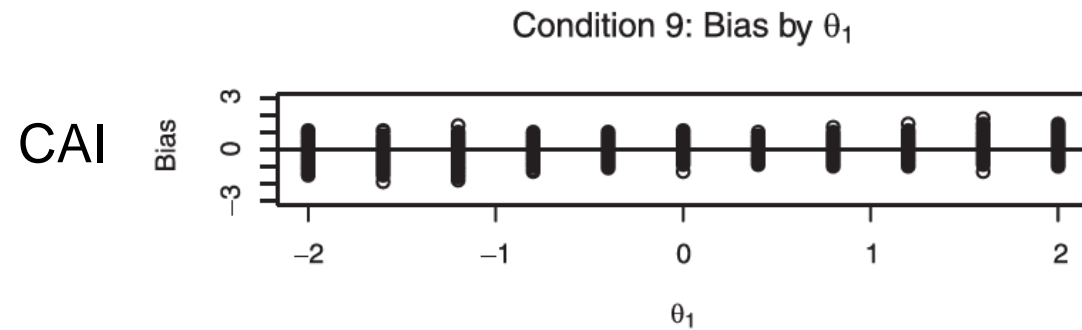
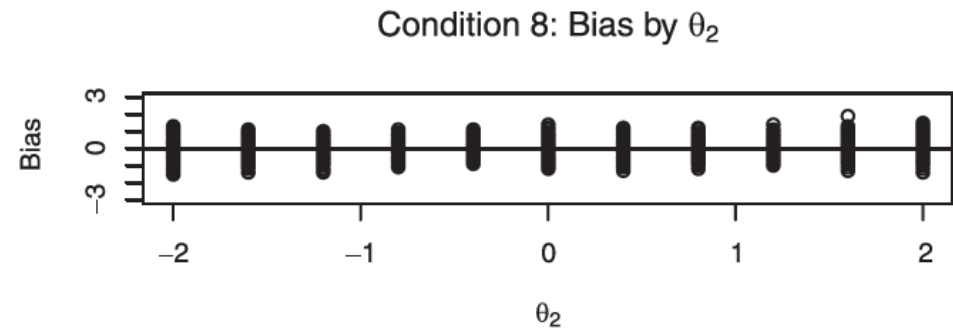
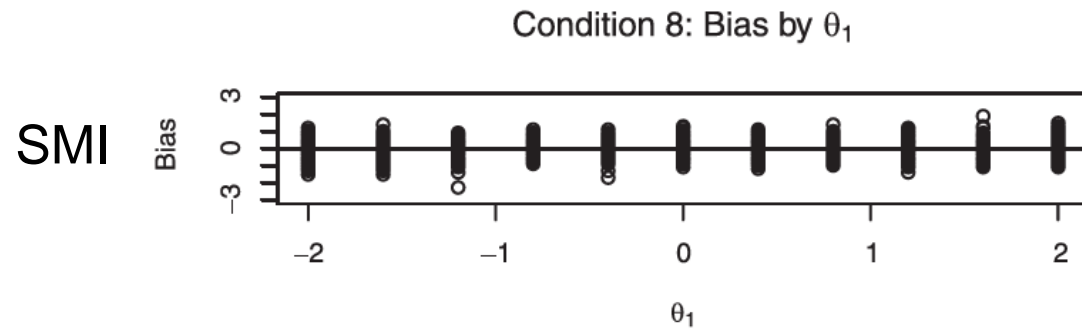
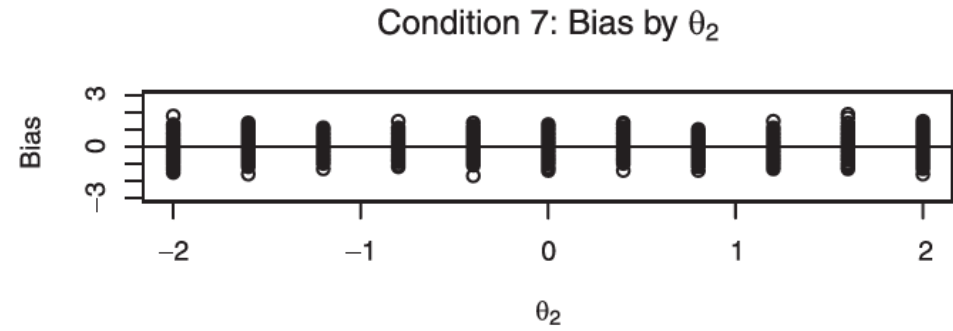
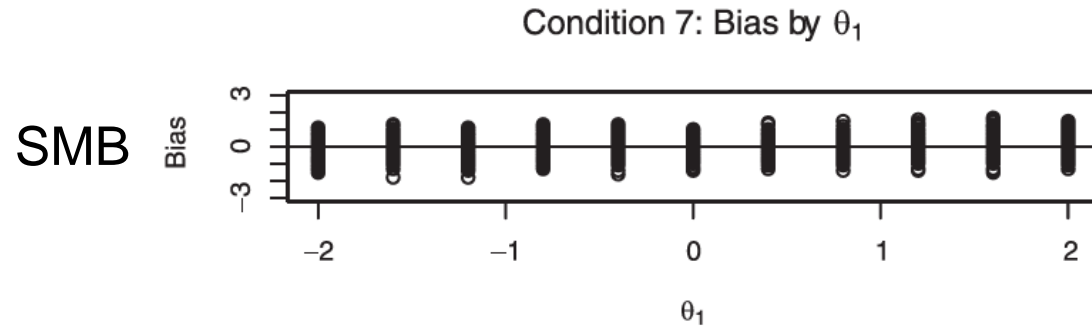


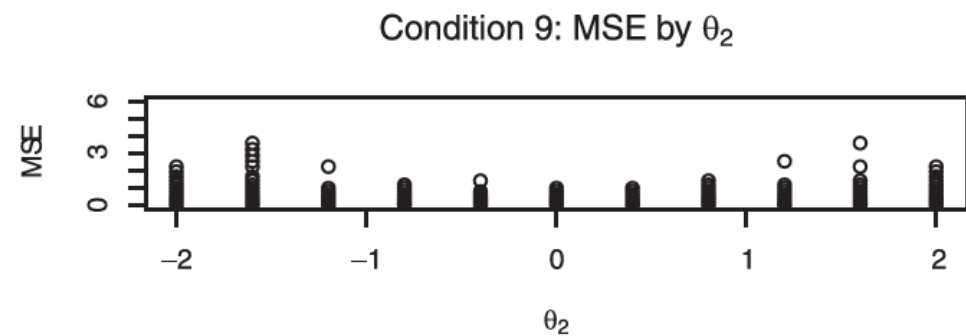
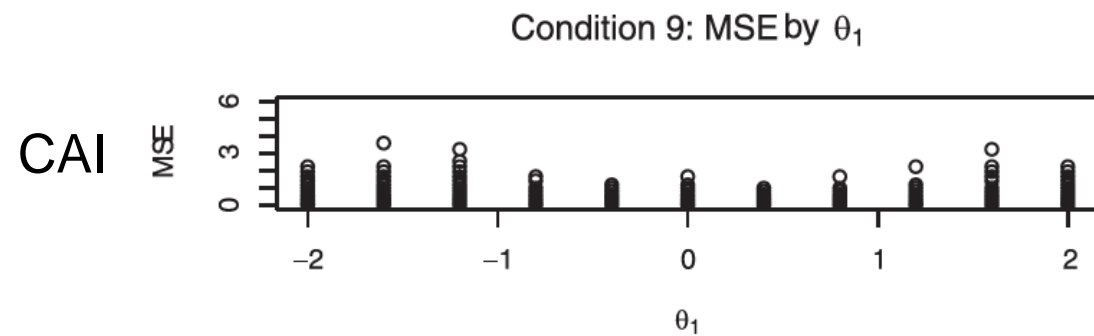
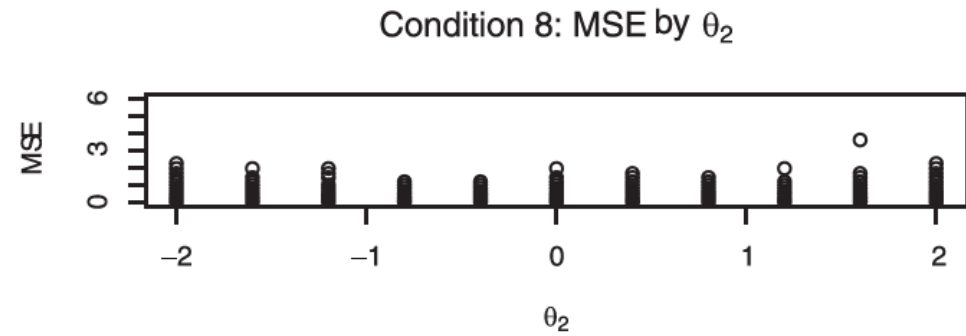
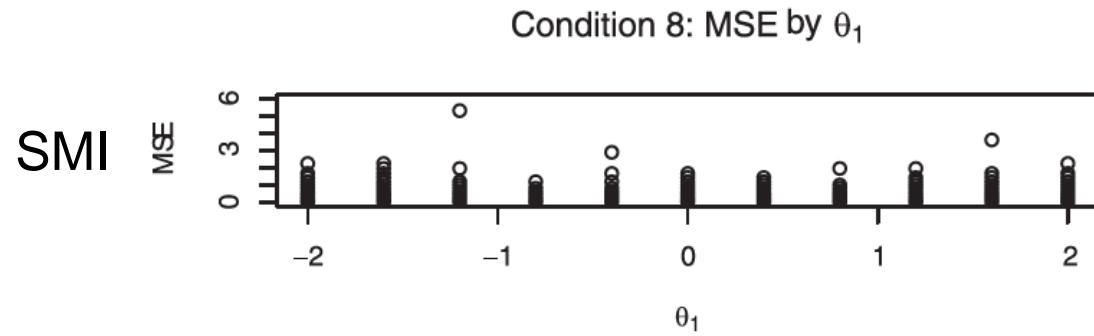
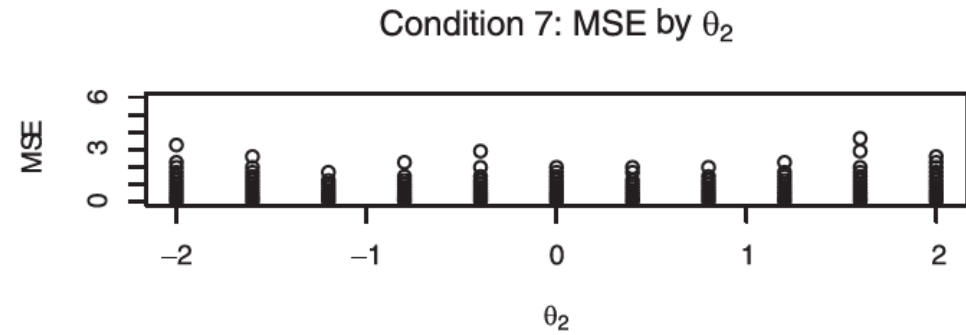
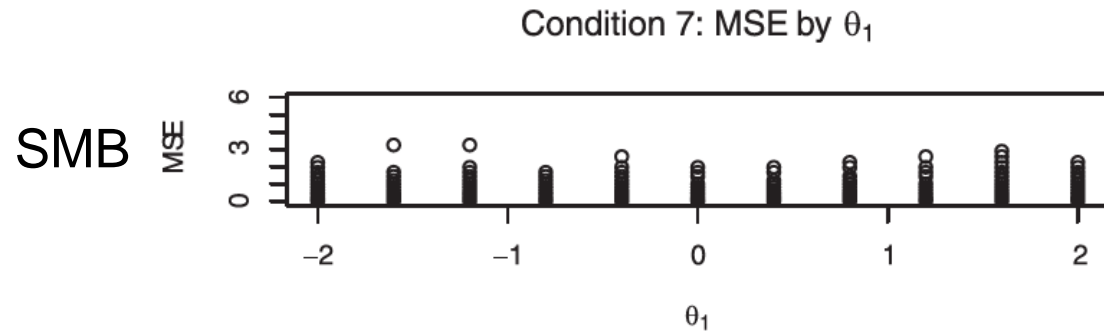
Condition	Method	Item exposure rates							
		<0.05	0.05-0.10	0.10-0.20	0.20-0.30	0.30-0.40	0.40-0.50	>0.50	
1	20 items	SMB	0.825	0.085	0.052	0.015	0.004	0.012	0.006
2		SMI	0.831	0.035	0.048	0.044	0.017	0.012	0.012
3		CAI	0.775	0.062	0.094	0.050	0.010	0.004	0.004
7	30 items	SMB	0.640	0.206	0.098	0.027	0.010	0.008	0.010
8		SMI	0.754	0.048	0.071	0.058	0.029	0.015	0.025
9		CAI	0.696	0.050	0.131	0.092	0.017	0.004	0.010

Note. SMB = a -stratification with match- b item selection; SMI = a -stratification with maximum Fisher information item selection; CAI = continuous a -stratification.

Condition	Length	Ability	Method	Bias	MSE	χ^2
1	40	BN ($\rho = .3$)	SMB	-0.003	0.37	12.87
2	40	BN ($\rho = .3$)	SMI	0.001	0.28	86.55
3	40	BN ($\rho = .3$)	CAI	-0.003	0.26	57.17
4	40	BN ($\rho = .7$)	SMB	-0.007	0.40	14.15
5	40	BN ($\rho = .7$)	SMI	0.000	0.30	81.63
6	40	BN ($\rho = .7$)	CAI	-0.005	0.27	58.61
7	40	Grid	SMB	0.002	0.39	16.77
8	40	Grid	SMI	-0.005	0.31	75.96
9	40	Grid	CAI	-0.008	0.28	51.55
10	60	BN ($\rho = .3$)	SMB	-0.001	0.24	8.88
11	60	BN ($\rho = .3$)	SMI	-0.002	0.19	84.83
12	60	BN ($\rho = .3$)	CAI	0.001	0.17	54.40
13	60	BN ($\rho = .7$)	SMB	-0.003	0.26	9.50
14	60	BN ($\rho = .7$)	SMI	-0.004	0.21	78.47
15	60	BN ($\rho = .7$)	CAI	-0.008	0.19	55.91
16	60	Grid	SMB	-0.004	0.26	13.08
17	60	Grid	SMI	-0.003	0.21	71.60
18	60	Grid	CAI	0.003	0.18	46.56

Note. MSE = mean squared error; BN = bivariate normal ability generating distribution with correlation ρ ; SMB = α -stratification with match- b item selection; SMI = α -stratification with maximum D-optimality item selection; CAI = continuous α -stratification.



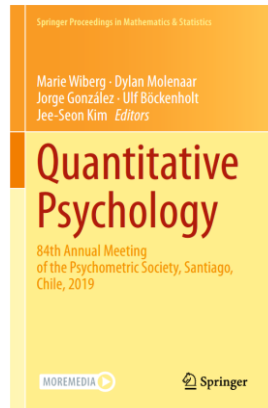


Condition	Method	Item exposure rates							
		<0.05	0.05-0.1	0.10-0.20	0.20-0.30	0.30-0.40	0.40-0.50	>0.50	
1	SMB	0.173	0.550	0.271	0.002	0.000	0.000	0.004	
2	SMI	0.640	0.069	0.106	0.102	0.060	0.019	0.004	
3	40 items	CAI	0.531	0.129	0.185	0.112	0.035	0.006	0.000
4		SMB	0.196	0.548	0.250	0.002	0.000	0.000	0.004
5		SMI	0.619	0.088	0.112	0.102	0.056	0.021	0.002
6		CAI	0.540	0.115	0.194	0.106	0.038	0.008	0.000
10		SMB	0.010	0.262	0.694	0.029	0.000	0.000	0.004
11	SMI	0.494	0.096	0.133	0.133	0.071	0.058	0.015	
12	60 items	CAI	0.373	0.135	0.235	0.154	0.075	0.021	0.006
13		SMB	0.015	0.296	0.654	0.031	0.000	0.000	0.004
14		SMI	0.477	0.090	0.152	0.140	0.077	0.050	0.015
15		CAI	0.381	0.131	0.221	0.165	0.077	0.021	0.004

Note. BN = bivariate normal ability generating distribution with correlation ρ ; SMB = α -stratification with match- b item selection; SMI = α -stratification with maximum D-optimality item selection; CAI = continuous α -stratification.

- The **SMB** method was shown to be the **best** by far in terms of **item exposure control** but yielded consistently **high MSE**.
- **CAI** was similar to or **better than that of SMI** in terms of bias and MSE while producing smaller χ^2 values.
- The manner in **which the test is started** may affect the performance of each method.

- Only one value was examined for β & only one item bank was simulated for each study & the number of strata was fixed at 4.
- An advantage of the CAI method that may not be readily apparent is its ability to be **extended to more than two dimensions**.
- CAI could be **combined with a method** of maximum exposure control to limit the maximum item exposure rate.
- **Relax** the fact that CAI always forces the discrimination parameters of the selected items to be **strictly ascending**.



[The Annual Meeting of the Psychometric Society](#)

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Performance of the Modified Continuous α -Stratification Indices in Computerized Adaptive Testing

Authors

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Ya-Hui Su

- the maximum item exposure rate: commonly set to 0.2

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Note. SMB = a -stratification with match- b item selection; SMI = a -stratification with maximum Fisher information item selection; CAI = continuous a -stratification.

➡ **Purpose:** combine the CAI method with item exposure control methods to limit item exposure

- the maximum item exposure rate: commonly set to 0.2

Condition	Method	<0.05	0.05-0.10	0.10-0.20	Item exposure rates			
					0.20-0.30	0.30-0.40	0.40-0.50	>0.50
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Note. SMB = a -stratification with match- b item selection; SMI = a -stratification with maximum Fisher information item selection; CAI = continuous a -stratification.

➡ **Purpose:** combine the CAI method with item exposure control methods to limit item exposure

- three modified CAI methods:
 - CAI + exposure
 - CAI + freeze
 - CAI + SHOF

- three modified CAI methods:

- CAI + exposure

- CAI + freeze

- CAI + SHOF

- maximum priority index

$$PI_j = Inf_i \times \prod_{k=1}^K (\omega_k f_k)^{c_{jk}}$$

$$f_k = \frac{1}{r_{\max}} \left(r_{\max} - \frac{s_i}{S} \right)$$

$$= \frac{1}{0.2} \left(0.2 - \frac{\text{examinees have seen item } i}{\text{examinees have taken the CAT}} \right)$$

➡ $CAI_i \times Inf_i \times \frac{\left(r_{\max} - \frac{s_i}{S} \right)}{r_{\max}}$

- three modified CAI methods:

- CAI + exposure
- CAI + freeze
- CAI + SHOF

make popular items appear
in a predictable sequence:
1, 6, 11, 16, ...

- maximum priority index

$$PI_j = Inf_i \times \prod_{k=1}^K (\omega_k f_k)^{c_{jk}}$$

$$f_k = \frac{1}{r_{\max}} \left(r_{\max} - \frac{S_i}{S} \right)$$

$$= \frac{1}{0.2} \left(0.2 - \frac{\text{examinees have seen item } i}{\text{examinees have taken the CAT}} \right)$$

➡ $CAI_i \times Inf_i \times \frac{\left(r_{\max} - \frac{S_i}{S} \right)}{r_{\max}}$

- ➤ freeze control
- $CAI_i \times Inf_i$
- ➡ if reach γ_{\max} ,
not included temporarily

- three modified CAI methods:

- CAI + exposure
- CAI + freeze
- CAI + SHOF

➤ The Sympson and Hetter online procedure with freeze (SHOF)

$p(S)$ → the probability that an item is **‘selected’**

$p(A)$ → the probability that an item is actually **‘administered’**

➡ $p(A) = p(A|S) \times p(S) \leq r_{\max}$

↓
to adjust $p(S)$ such that $p(A)$ is less than or equal to r_{\max}

a random number is less than $p(A|S)$: administer
otherwise: select next item



a series of iterative simulations is needed to find stabilized $p(A|S)$

- The Simpson and Hetter online procedure with freeze (SHOF)

$$p(A) = p(A|S) \times p(S) \leq r_{\max}$$

How to update $p(A|S)$ sequentially during the test?

CAI + SHOF

Step 1. Set the initial $p(A|S) = 1$ for all items in the item pool.

Step 2. Administer CATs to the j th examinee as in the SH procedure.

Step 3. Find $p(S)$ and $p(A)$ for each item by computing the proportion of times an item has been selected and administered, respectively.

Step 4. Update $p(A|S)$ for each item based on $p(S)$ and $p(A)$ as follows:

if $p(A) > r_{\max}$, then $p(A|S) = 0.0$;

if $p(A) \leq r_{\max}$ and $p(S) > r_{\max}$, then $p(A|S) = r_{\max} / p(S)$;

if $p(A) \leq r_{\max}$ and $p(S) \leq r_{\max}$, then $p(A|S) = 1.0$

Step 5. Repeat step 2 until CATs have been administered to all examinees.

- Item selection method
 - **CAI** method ($\beta = 2$)
 - a-stratification method ($K = 4$):
SMI: Maximizing Fisher information & **SMB**: Match-b
 - three **modified CAI** methods: CAI + exposure, CAI + freeze, and CAI + SHOF
- Item pool structure
 - test length: 20 & 30 items
 - item bank: 500 items
 - $a \sim U(0.0, 1.3)$
 - $b \sim U(-1.3, 1.3)$
 - $c \sim U(0.2, 0.3)$
- Examinee generation
 - examinee abilities:
normal(0,1) distribution: 5,100 examinees
 - theta estimation:
maximum likelihood estimator (MLE)

- Evaluation Criteria

- measurement precision

- bias = $\frac{1}{N} \sum_{n=1}^N (\hat{\theta}_n - \theta_n)$

- RMSE = $\sqrt{\frac{1}{N} \sum_{n=1}^N (\hat{\theta}_n - \theta_n)^2}$

- $r_{\theta, \hat{\theta}} = \frac{COV_{\theta, \hat{\theta}}}{S_{\theta} S_{\hat{\theta}}}$

- relative efficiency (RE) = $\frac{RMSE_{SMI}}{RMSE_{others}}$

- exposure control

- $\chi^2 = \frac{1}{L/I} \sum_{i=1}^I (r_i - L/I)^2$

Table 1 Measurement precision of the six item selection methods under various conditions

Item length	Item selection methods	bias	RMSE	RE	$r_{\theta, \hat{\theta}}$
20	SMB	0.068	0.575	0.858	0.864
	SMI	0.024	0.494	1.000	0.901
	CAI	0.018	0.469	1.053	0.912
	CAI + exposure	0.026	0.496	0.996	0.901
	CAI + freeze	0.016	0.461	1.072	0.916
	CAI + SHOF	0.020	0.469	1.054	0.912
30	SMB	0.018	0.397	0.923	0.934
	SMI	-0.001	0.366	1.000	0.944
	CAI	0.007	0.363	1.009	0.943
	CAI + exposure	0.016	0.402	0.911	0.933
	CAI + freeze	0.012	0.361	1.013	0.944
	CAI + SHOF	0.011	0.372	0.983	0.940

Table 2 Exposure control of the six item selection methods under various conditions

Test length	Item selection methods	Item exposure rates								Max.	Chi-square statistics
		0	0–0.05	0.05–0.1	0.1–0.2	0.2–0.3	0.3–0.4	0.4–0.5	>0.5		
20	SMB	0.036	0.874	0.012	0.006	0.028	0.026	0.012	0.006	0.79	116.67
	SMI	0.724	0.078	0.044	0.082	0.046	0.012	0.004	0.010	0.81	125.65
	CAI	0.680	0.106	0.070	0.076	0.040	0.016	0.008	0.004	0.78	104.51
	CAI+exposure	0.058	0.610	0.306	0.026	–	–	–	–	0.13	9.64
	CAI+freeze	0.570	0.164	0.090	0.176	–	–	–	–	0.20	56.22
	CAI+SHOF	0.574	0.154	0.082	0.190	–	–	–	–	0.20	54.34
30	SMB	0.028	0.768	0.084	0.006	0.024	0.072	0.012	0.006	0.80	109.59
	SMI	0.658	0.066	0.062	0.084	0.068	0.040	0.012	0.010	0.81	117.84
	CAI	0.600	0.086	0.084	0.116	0.060	0.038	0.006	0.010	0.60	93.97
	CAI+exposure	0.028	0.408	0.432	0.132	–	–	–	–	0.14	7.53
	CAI+freeze	0.436	0.186	0.084	0.294	–	–	–	–	0.20	51.30
	CAI+SHOF	0.430	0.188	0.082	0.300	–	–	–	–	0.20	48.75

- CAI + exposure method showed **great potential for monitoring item exposure**; however, it did not have the best measurement precision.
- Satisfy all the **constraints simultaneously** during item selection.
- The idea of the modified CAI methods can easily be extended to **multidimensional contexts** for item selection.
- The efficiency of item selection methods with different item exposure methods or **test overlap methods**.

Thanks for listening!

Reporter: Yingshi Huang

谢谢大家
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预祝新年快乐
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:D

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