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USING RESPONSE TIMES AND RESPONSE ACCURACY TO MEASURE FLUENCY WITHIN COGNITIVE DIAGNOSIS MODELS

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Introduction

- Cognitive diagnosis model (CDM)
  - Identifying a profile of strengths and weakness on assessed skills
  - Mastery: skills that are completed with accuracy

Mastery = accuracy in a skill + complete the task with fluency

The 2PLM ???

$$P_j(\theta_i) = \frac{\exp(Da_j(\theta_i - b_j))}{1 + \exp(Da_j(\theta_i - b_j))}$$

The GRM & NRM ???

$$P^*_{ix}(\theta) = \frac{\exp[Da_i(\theta - b_{ix})]}{1 + \exp[Da_i(\theta - b_{ix})]}$$

$$P_{ix}(\theta) = P^*_{ix}(\theta) - P^*_{i,x+1}(\theta)$$

$$P_{ix}(\theta) = \frac{\exp(c_{ix} + a_{ix}\theta)}{\sum_{h=1}^{n_i} \exp(c_{ih} + a_{ih}\theta)}$$

Introduction

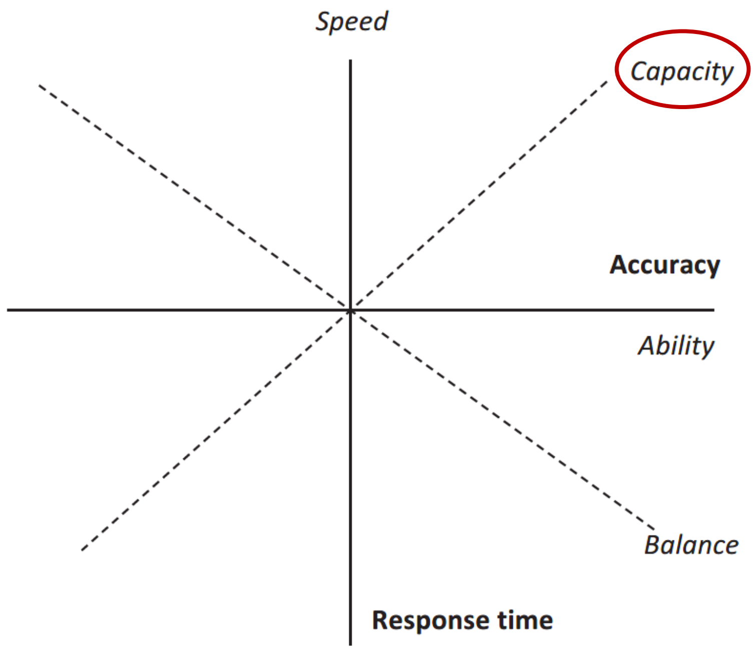
- Cognitive diagnosis model (CDM)
  - Identifying a profile of strengths and weakness on assessed skills
  - Mastery: skills that are completed with accuracy

Mastery = accuracy in a skill + complete the task with fluency

- What is fluency?
  - Speed: spontaneous speed & imposed speed

+
accuracy

- The aim of this study:
  - further advance the current CDM to **measure fluency directly**



[De Boeck et al. 2017 *BJMSP*]

Introduction

- Response accuracy
  - latent trait models
  - CDM: categorical trait, 1 indicating accuracy in the skill and 0 indicating otherwise

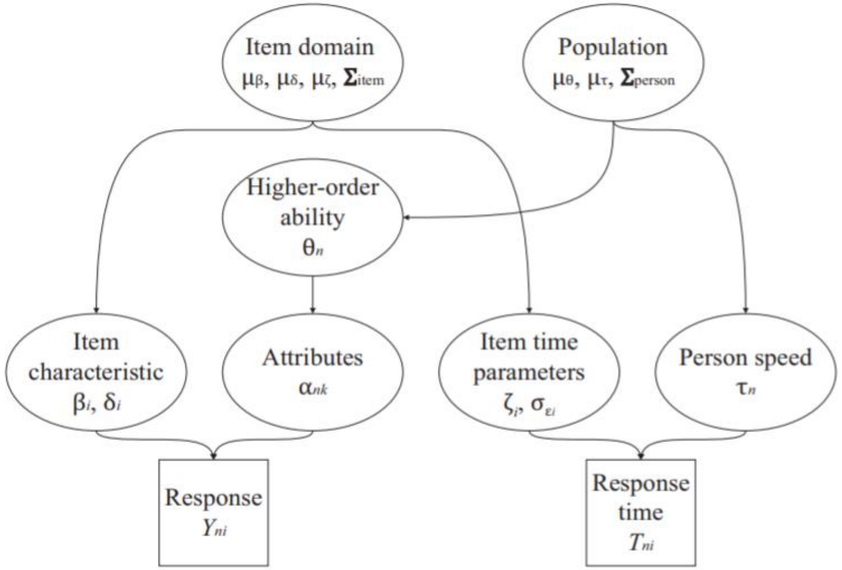
- Response time
  - pure response time models $T_{pi} \leftarrow$
  - joint models for response times and other variables $[T_{pi}, A_{pi}] \leftarrow$
  - local dependency models $([T_{pi} \leftrightarrow A_{pi}]) \leftarrow$
  - response time as covariate $A_{pi} \leftarrow T_{pi}$

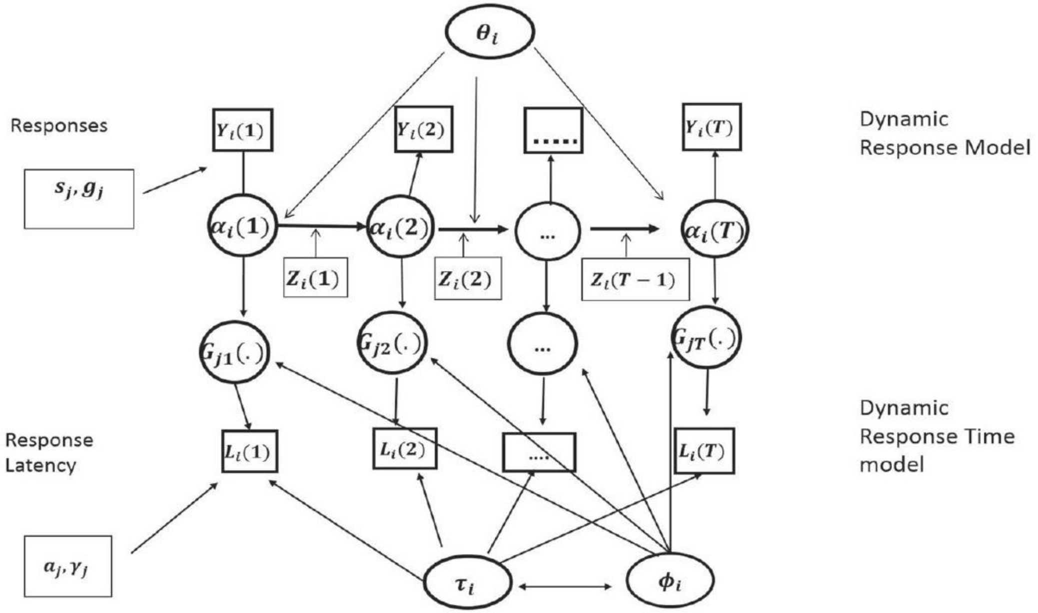
➡ within the IRT framework

De Boeck & Jeon, 2019 *Front Psychol*

Introduction

- CDM & response time
  - Zhan et al. (2018) & Wang et al. (2020, 2018b, 2019)





utilizing response times as ancillary information to improve the measurement accuracy

CDM for Fluency

- How to directly measure the fluency?

1. a specific relationship between accuracy and speed

   → speed tends to improve as accuracy reaches some level of proficiency

2. an automatic process

   → a fast, correct response represents a higher skill level

3. **defining fluency as the highest level of a polytomous CDM**

   → three attribute levels:

   $\alpha_{ik} = 0$: low accuracy

   $\alpha_{ik} = 1$: high response accuracy but slow speed on correct answers

   $\alpha_{ik} = 2$: high response accuracy and fast speed on correct answers

CDM for Fluency

- Response accuracy
  - J test questions to measure K latent skills
  - N test takers
  - response time: L_{ij}
  - response accuracy: Y_{ij}
  - Q matrix for item j : $\mathbf{q}_j = (q_{j1}, \ldots, q_{jK})^T$
  - latent attribute for examinee i : $\boldsymbol{\alpha}_i = (\alpha_{i1}, \ldots, \alpha_{iK})^T$

➡

a conjunctive model: $\eta_{ij} := \eta_{ij}(\boldsymbol{\alpha}_i, \mathbf{q}_j) = 1_{\{\forall k, q_{jk}=1, \alpha_{ik}=q_{jk}\}} + 1_{\{\forall k, q_{jk}=1, \alpha_{ik}>q_{jk}\}}$

$$P(Y_{ij}=1|\boldsymbol{\alpha}_i) = \begin{cases} g_j, & \text{if } \eta_{ij}=0 \\ 1-s_{1j}, & \text{if } \eta_{ij}=1 \\ 1-s_{2j}, & \text{if } \eta_{ij}=2 \end{cases} \quad \text{with} \quad 0 < g_j < 1-s_{1j} < 1-s_{2j} < 1 \qquad (1)$$

CDM for Fluency

- Response time
  - **a conditional model:**
    - → consider response times on correct responses only
    - → avoid the confounding influence from the fast but inaccurate responses

same base speed

□ $$\log(L_{ij}) \sim \begin{cases} N(\gamma_j - (\tau_i + \phi_i \times g(\boldsymbol{\alpha}_i, \mathbf{q}_j)), \frac{1}{a_j}) & \text{if } Y_{ij} = 1 \\ N(\gamma_j - \tau_i, \frac{1}{a_j}) & \text{if } Y_{ij} = 0 \end{cases}, \quad \tau_i \sim N(\mu_\tau, \sigma_\tau^2) \qquad (2)$$

individual speed change parameter > 0

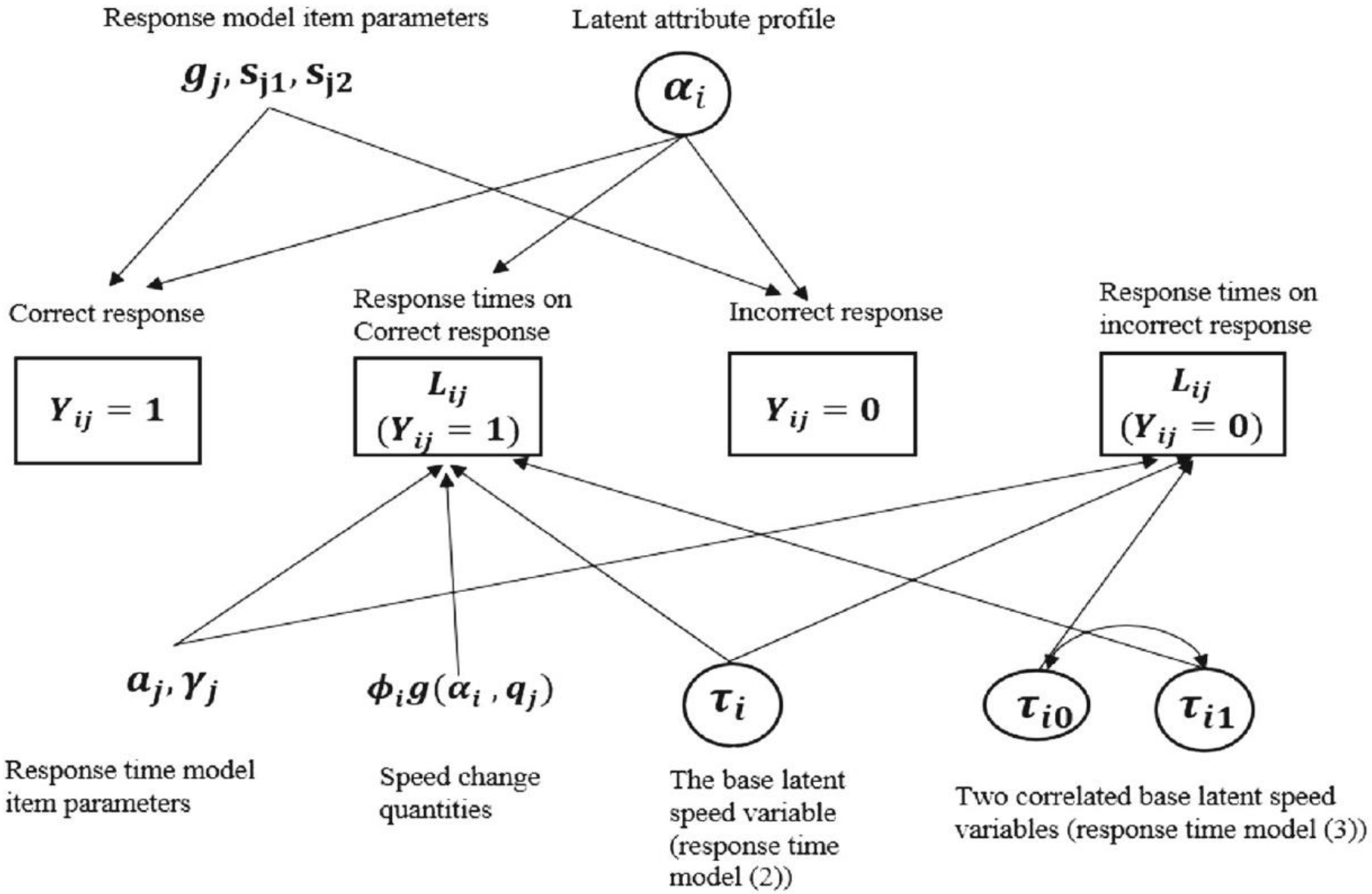
monotonic function that represents the influence from $\boldsymbol{\alpha}_i$

$$g(\boldsymbol{\alpha}_i, \mathbf{q}_j) = \begin{cases} \boldsymbol{\alpha}_i^T \mathbf{q}_j = \sum_{k=1}^K a_{ik} q_{jk}, \\ \eta_{ij}, \\ 1_{\{\eta_{ij}=2\}}. \end{cases}$$

different base speeds

□ $$\log(L_{ij}) \sim \begin{cases} N(\gamma_j - (\tau_{i1} + \phi_i \times g(\boldsymbol{\alpha}_i, \mathbf{q}_j)), \frac{1}{a_j}) & \text{if } Y_{ij} = 1 \\ N(\gamma_j - \tau_{i0}, \frac{1}{a_j}) & \text{if } Y_{ij} = 0 \end{cases}, \quad (\tau_{0i}, \tau_{1i}) \sim N(\boldsymbol{\mu}, \Sigma_{\tau_0 \tau_1}) \qquad (3)$$

Model Summary



Model Estimation

- Likelihood function
  - for the joint model using response time model 2:

$$L(\Omega|Y, L) = \prod_{j=1}^{J}\prod_{i=1}^{N} f(Y_{ij}|\boldsymbol{\alpha}_i, s_{1j}, s_{2j}, g_j) f(L_{ij}|Y_{ij}, \boldsymbol{\alpha}_i, \tau_i, \gamma_j, a_j, \phi_i) f(\tau_i|\sigma_\tau^2)$$

$f(Y_{ij}|\boldsymbol{\alpha}_i, s_{1j}, s_{2j}, g_j)$: $P(Y_{ij} = 1|\boldsymbol{\alpha}_i)^{Y_{ij}}(1 - P(Y_{ij} = 1|\boldsymbol{\alpha}_i))^{1-Y_{ij}}$

$f(L_{ij}|Y_{ij}, \boldsymbol{\alpha}_i, \tau_i, \gamma_j, a_j, \phi_i)$:

mean: $\gamma_j - (\tau_i + \phi_i \times g(\boldsymbol{\alpha}_i, \mathbf{q}_j))$ $Y_{ij} = 1$

$\gamma_j - \tau_i$ $Y_{ij} = 0$

variance: $\frac{1}{a_j}$

$f(\tau_i|\sigma_\tau^2)$: mean: 0, variance: σ_τ^2

  - for the joint model using response time model 3:

$$L(\Omega|Y, L) = \prod_{j=1}^{J}\prod_{i=1}^{N} f(Y_{ij}|\boldsymbol{\alpha}_i, s_{1j}, s_{2j}, g_j) f(L_{ij}|Y_{ij}, \boldsymbol{\alpha}_i, \tau_{i0}, \tau_{i1}, \gamma_j, a_j, \phi_i) f(\tau_{i0}, \tau_{i1}|\Sigma_{\tau_0\tau_1})$$

$f(L_{ij}|Y_{ij}, \boldsymbol{\alpha}_i, \tau_{i0}, \tau_{i1}, \gamma_j, a_j, \phi_i)$:

mean: $\gamma_j - (\tau_{i1} + \phi_i \times g(\boldsymbol{\alpha}_i, \mathbf{q}_j))$ $Y_{ij} = 1$

$\gamma_j - \tau_{i0}$ $Y_{ij} = 0$

$f(\tau_{i0}, \tau_{i1}|\Sigma_{\tau_0\tau_1})$: $(0, 0)^T$, $\Sigma_{\tau_0\tau_1}$

Model Estimation

- Following a principle of **conjugate priors**
  - take the variance of speed for model 2 as an example

    $\tau_i \sim N(\mu_\tau, \sigma_\tau^2)$ (restrict $\mu_\tau = 0$ to fix the location of the latent continuous variables)

  - the selection of conjugate prior is determintered by **the kernel of the likelihood function**:

$$p(\boldsymbol{\tau}|\sigma_\tau^2) = (\frac{1}{\sqrt{2\pi}\sigma_\tau})^N \exp\left\{-\frac{1}{2\sigma_\tau^2}\sum_{i=1}^{N}(\tau_i - 0)^2\right\} \propto (\frac{1}{\sigma_\tau^2})^{N/2}\exp\{-\frac{1}{2\sigma_\tau^2}\sum_{i=1}^{N}(\tau_i - 0)^2\}$$

$$Inv - Gamma(x; a, b) = \frac{b^a}{\Gamma(a)}(\frac{1}{x})^{a+1}\exp(-\frac{1}{x}b)$$

the prior: $\pi(\sigma_\tau^2) = Inv - Gamma(\sigma_\tau^2; a, b)$

$$p(\sigma_\tau^2|\boldsymbol{\tau}) \propto p(\boldsymbol{\tau}|\sigma_\tau^2)\pi(\sigma_\tau^2) \propto \left(\frac{1}{\sigma_\tau^2}\right)^{a+1+\frac{N}{2}}\exp\{-\frac{1}{\sigma_\tau^2}(b + \frac{1}{2}\sum_{i=1}^{N}\tau_i^2)\}$$

$\Rightarrow$

$$Inv - Gamma\left(\sigma_\tau^2; a + \frac{N}{2}, b + \frac{1}{2}\sum_{i=1}^{N}\tau_i^2\right)$$

Model Estimation

$$L(\Omega|Y, L) = \prod_{j=1}^{J}\prod_{i=1}^{N} f(Y_{ij}|\boldsymbol{\alpha}_i, s_{1j}, s_{2j}, g_j) f(L_{ij}|Y_{ij}, \boldsymbol{\alpha}_i, \tau_i, \gamma_j, a_j, \phi_i) f(\tau_i|\sigma_\tau^2)$$

- The full conditional distributions

Response model (1)

$s_{j2}\|s_{j1}$	$\text{Beta}(\tilde{\alpha}_{s2}, \tilde{\beta}_{s2})1_{\{s_{j1}>s_{j2}\}}$,
	$\tilde{\alpha}_{s2} = \sum_{i:\eta_{ij}=2}(1 - Y_{ij}) + \alpha_{s2}$, and $\tilde{\beta}_{s2} = \sum_{i:\eta_{ij}=2} Y_{ij} + \beta_{s2}$
$s_{j1}\|s_{j2}, g_j$	$\text{Beta}(\tilde{\alpha}_{s1}, \tilde{\beta}_{s1})1_{\{s_{j2}<s_{j1}<1-g_j\}}$
	$\tilde{\alpha}_{s1} = \sum_{i:\eta_{ij}=1}(1 - Y_{ij}) + \alpha_{s1}$, and $\tilde{\beta}_{s1} = \sum_{i:\eta_{ij}=1} Y_{ij} + \beta_{s1}$
$g_j\|s_{j1}$	$\text{Beta}(\tilde{\alpha}_g, \tilde{\beta}_g)1_{\{0\le g_j<1-s_{j1}\}}$
	$\tilde{\alpha}_g = \sum_{i:\eta_{ij}=0} Y_{ij} + \alpha_g$, and $\tilde{\beta}_g = \sum_{i:\eta_{ij}=0}(1 - Y_{ij}) + \beta_g$
$\boldsymbol{\alpha}_i$	Multinominal distribution with 3^K categories and parameter $\tilde{\pi}_{ic}$
	$\tilde{\pi}_{ic} = \frac{P(Y_i\|\boldsymbol{\alpha}_i=\boldsymbol{\alpha}_c,\Omega) f(L_i\|\boldsymbol{\tau},\phi_i,\boldsymbol{\alpha}_i=\boldsymbol{\alpha}_c)\pi_c}{\sum_{c=1}^{C} P(Y_i\|\boldsymbol{\alpha}_i=\boldsymbol{\alpha}_c,\Omega) f(L_i\|\boldsymbol{\tau},\phi,\boldsymbol{\alpha}_i=\boldsymbol{\alpha}_c)\pi_c}$
$\boldsymbol{\pi}$	$\text{Dirichlet}(\tilde{\mathbf{N}} + \boldsymbol{\delta}_0)$
	$\tilde{\mathbf{N}} = (\tilde{N}_1, \ldots \tilde{N}_C)$, and $\tilde{N}_c = \sum_{i=1}^{N} 1_{(\boldsymbol{\alpha}_i=\boldsymbol{\alpha}_c)}$

Model Estimation

$$L(\Omega|Y, L) = \prod_{j=1}^{J}\prod_{i=1}^{N} f(Y_{ij}|\boldsymbol{\alpha}_i, s_{1j}, s_{2j}, g_j) f(L_{ij}|Y_{ij}, \boldsymbol{\alpha}_i, \tau_i, \gamma_j, a_j, \phi_i) f(\tau_i|\sigma_\tau^2)$$

- The full conditional distributions

$$\log(L_{ij}) \sim \begin{cases} N(\gamma_j - (\tau_i + \phi_i \times g(\boldsymbol{\alpha}_i, \mathbf{q}_j)), \frac{1}{a_j}) & \text{if } Y_{ij} = 1 \\ N(\gamma_j - \tau_i, \frac{1}{a_j}) & \text{if } Y_{ij} = 0 \end{cases}, \quad \tau_i \sim N(\mu_\tau, \sigma_\tau^2)$$

Response time model (2)

a_j^2	$\text{Gamma}\left(a_1 + \frac{N}{2}, b_1 + \frac{1}{2}\sum_{i=1}^{N}(log(L_{ij}) + \tau_i + \phi_i \cdot g(\boldsymbol{\alpha}_i, \mathbf{q}_j)1_{\{Y_{ij}=1\}} - \gamma_j)^2\right).$
γ_j	$N\left(\frac{a_j^2 \cdot \sum_{i=1}^{N}\left(log(L_{ij})+\tau_i+\phi_i \cdot g(\boldsymbol{\alpha}_i,\mathbf{q}_j)1_{\{Y_{ij}=1\}}\right)}{Na_j^2 p_{\sigma_\gamma^2}+1}, \frac{p_{\sigma_\gamma^2}}{Na_j^2 p_{\sigma_\gamma^2}+1}\right)$
τ_i	$N\left(\frac{-\sum_j\left((\log L_{ij}-\gamma_j+\phi_i \cdot g(\boldsymbol{\alpha}_i,\mathbf{q}_j)1_{\{Y_{ij}=1\}}) \cdot a_j^2\right)}{\sum_j a_j^2+1/\sigma_\tau^2}, 1/(\sum_j(a_j)^2 + 1/\sigma_\tau^2)\right)$
σ_τ^2	$\text{Inv-Gamma}(a_2 + \frac{N}{2}, b_2 + \frac{\sum_{i=1}^{N}\tau_i^2}{2})$
ϕ_i	$N\left(-\frac{\sum_{j:Y_{ij}=1}[a_j^2 g(\boldsymbol{\alpha}_i,\mathbf{q}_j)(\log(L_{ij})-\gamma_j+\tau_i)]}{\sum_{j:Y_{ij}=1}[a_j^2 g^2(\boldsymbol{\alpha}_i,\mathbf{q}_j)]+1/\sigma_\phi^2}, \{\sum_{j=1}^{J}\sum_{i:Y_{ij}=1}[a_j^2 g^2(\boldsymbol{\alpha}_i, \mathbf{q}_j)] + 1/\sigma_\phi^2\}^{-1}\right)$

Model Estimation

$$L(\Omega|Y, L) = \prod_{j=1}^{J}\prod_{i=1}^{N} f(Y_{ij}|\boldsymbol{\alpha}_i, s_{1j}, s_{2j}, g_j) f(L_{ij}|Y_{ij}, \boldsymbol{\alpha}_i, \tau_{i0}, \tau_{i1}, \gamma_j, a_j, \phi_i) f(\tau_{i0}, \tau_{i1}|\Sigma_{\tau_0\tau_1})$$

$$\log(L_{ij}) \sim \begin{cases} N(\gamma_j - (\tau_{i1} + \phi_i \times g(\boldsymbol{\alpha}_i, \mathbf{q}_j)), \frac{1}{a_j}) & \text{if } Y_{ij} = 1 \\ N(\gamma_j - \tau_{i0}, \frac{1}{a_j}) & \text{if } Y_{ij} = 0 \end{cases}, \quad (\tau_{0i}, \tau_{1i}) \sim N(\boldsymbol{\mu}, \Sigma_{\tau_0\tau_1})$$

- The full conditional distributions

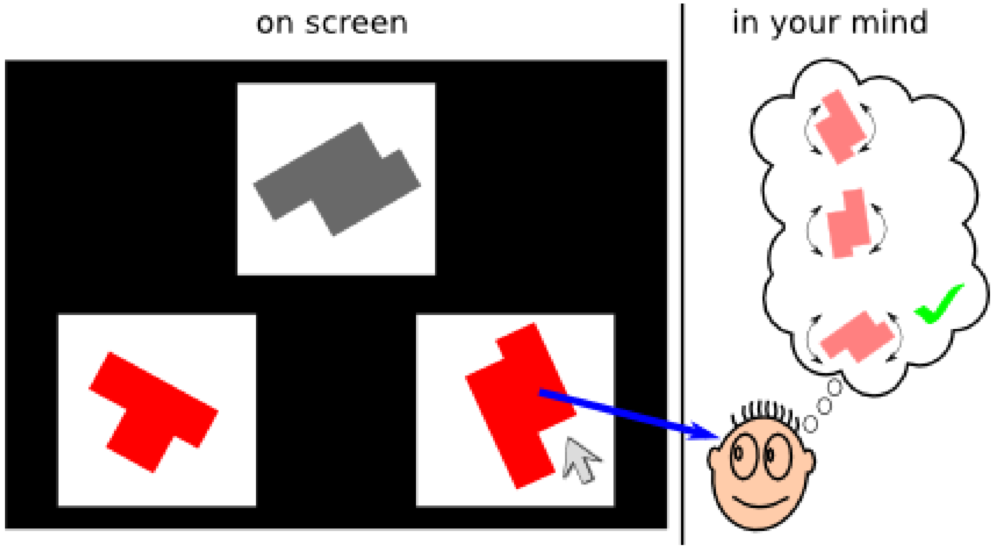
Response time model (3)

a_j^2		$\text{Gamma}\left(a_1 + \frac{N}{2}, b_1 + \frac{1}{2}\sum_{i=1}^{N}(log(L_{ij}) + (\tau_{1i} + \phi_i \cdot g(\boldsymbol{\alpha}_i, \mathbf{q}_j))1_{\{Y_{ij}=1\}} + \tau_{0i} \cdot 1_{\{Y_{ij}=0\}} - \gamma_j)^2\right)$
γ_j		$N\left(\frac{a_j^2 \cdot [\sum_{i=1}^{N}(log(L_{ij})) + \sum_{i:Y_{ij}=1}(\phi_i \cdot g(\boldsymbol{\alpha}_i, \mathbf{q}_j) + \tau_{1i}) + \sum_{i:Y_{ij}=0}\tau_{0i}]}{p_{\sigma_\gamma^2} N a_j^2 + 1}, \frac{p_{\sigma_\gamma^2}}{N a_j^2 p_{\sigma_\gamma^2} + 1}\right)$
τ_{0i}		$N\left(\frac{-\sum_{j:Y_{ij}=0}(\log L_{ij} - \gamma_j)(a_j)^2}{\sum_{j:Y_{ij}=0} a_j^2 + 1/\sigma_{\tau_0}^2}, 1/(\sum_{j:Y_{ij}=0} a_j^2 + 1/\sigma_{\tau_0}^2)\right)$
τ_{1i}		$N\left(\frac{-\sum_{j:Y_{ij}=1}(\log L_{ij} - \gamma_j + \phi_i \cdot g(\boldsymbol{\alpha}_i, \mathbf{q}_j))(a_j)^2}{\sum_{j:Y_{ij}=1} a_j^2 + 1/\sigma_{\tau_1}^2}, 1/(\sum_{j:Y_{ij}=1} a_j^2 + 1/\sigma_{\tau_1}^2)\right)$
ϕ_i		$N\left(-\frac{\sum_{j:Y_{ij}=1}[a_j^2 g(\boldsymbol{\alpha}_i, \mathbf{q}_j)(\log(L_{ij}) - \gamma_j + \tau_{1i})]}{\sum_{j:Y_{ij}=1}[a_j^2 g^2(\boldsymbol{\alpha}_i, \mathbf{q}_j)] + 1/\sigma_\phi^2}, \{\sum_{j:Y_{ij}=1}[a_j^2 g^2(\boldsymbol{\alpha}_i, \mathbf{q}_j)] + 1/\sigma_\phi^2\}^{-1}\right)$

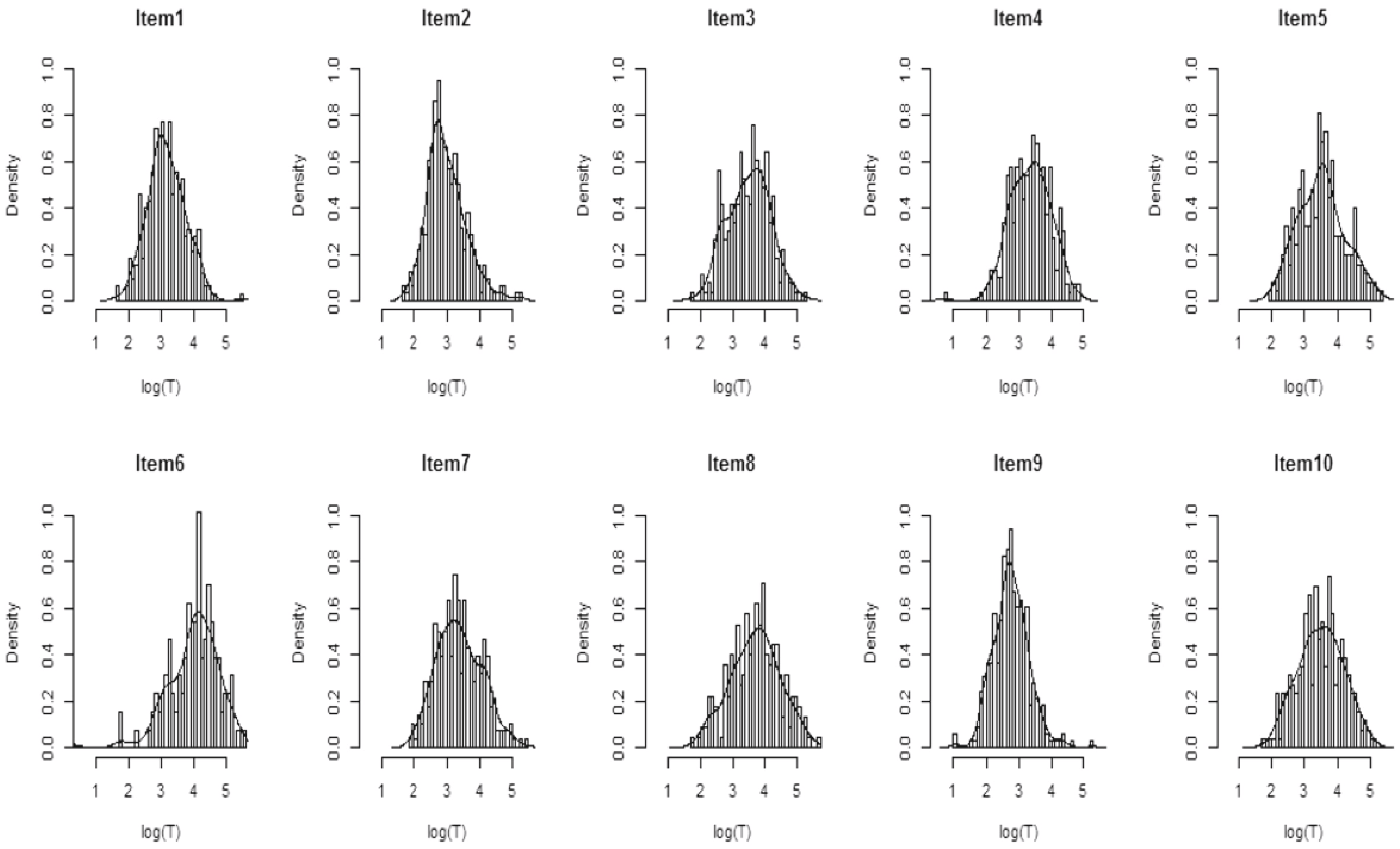
Real Data Application

- a computer-based mental rotation learning program
  - 351 participants' response times and responses to 10 items
  - four attributes were measured:

    α_1(90° x-axis), α_2(90° y-axis), α_3(180° x-axis) and α_4(180° y-axis)



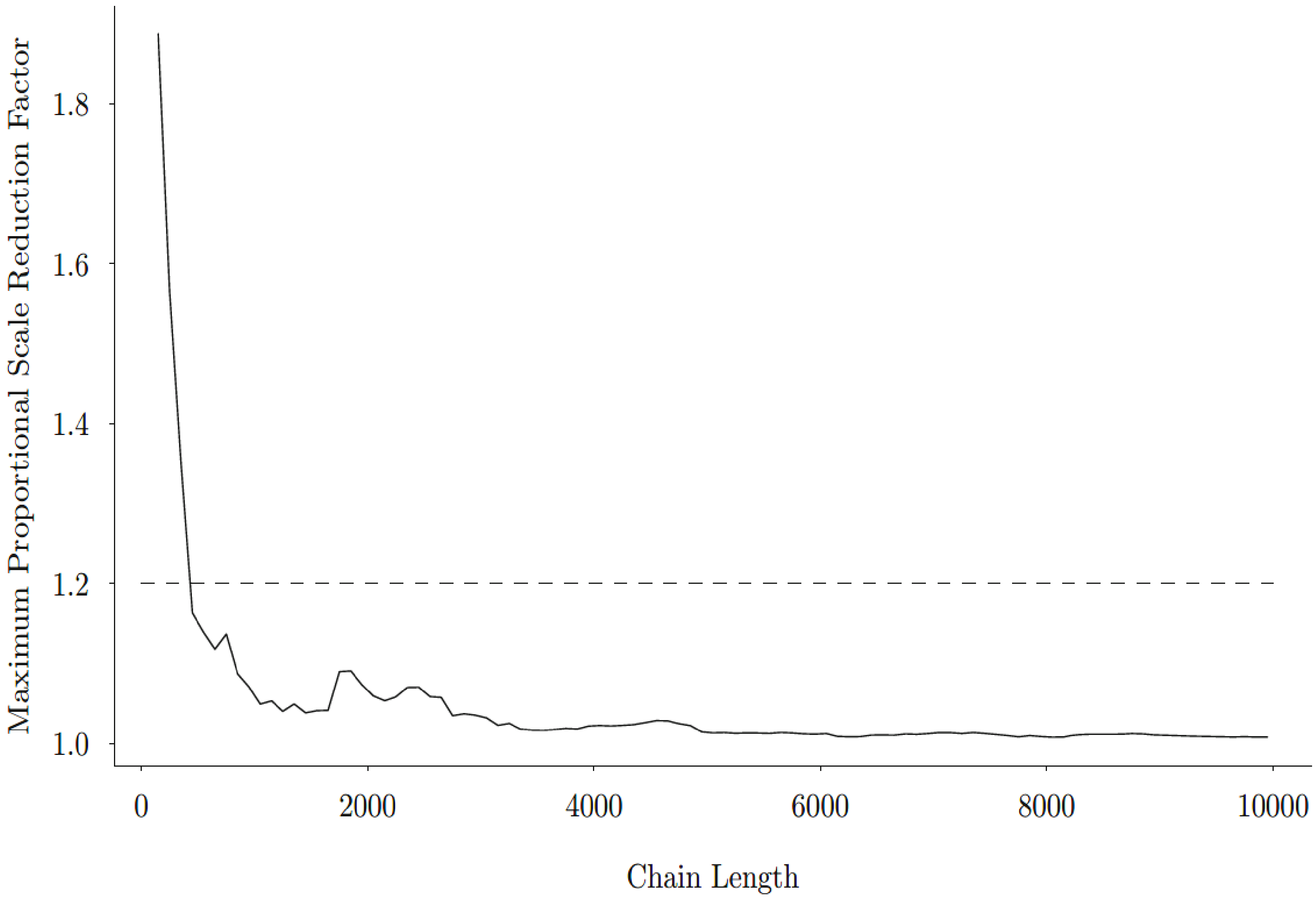
The Distribution of Log (Response Time) of Correct Response



Model Selections

- CDM fluency models:
  - response accuracy: model 1
  - response time model: model 2 or model 3
  - covariate function $g(\boldsymbol{\alpha}_i, \mathbf{q}_j)$: η_{ij} or $1_{\{\eta_{ij}=2\}}$
- baseline models:
  1. DINA model for responses and lognormal model for response times
  2. Model 1 fitted to only responses
- evaluation criteria:
  - model convergence: PSRF
  - model comparison: the deviance information criteria (DIC)

The PSRF for the Most Complicated Model



The Deviance Information Criteria

DIC of six models

Model				DIC		
Fluency	Response	RT	$g(\boldsymbol{\alpha}_i, \mathbf{q}_j)$	Response	RT	Joint
1	Eq. (1)	Eq. (2)	η_{ij}	2952.00	30,159.12	33,111.12
2	Eq. (1)	Eq. (2)	$1_{\{\eta_{ij}=2\}}$	2756.24	29,981.68	32,737.92
3	Eq. (1)	Eq. (3)	η_{ij}	2968.41	30,197.25	33,165.67
4	Eq. (1)	Eq. (3)	$1_{\{\eta_{ij}=2\}}$	2815.62	30,010.56	32,826.18
			Baseline			
1	DINA	Lognormal	–	3019.20	30,012.51	33,031.71
2	Eq. (1)	–	–	2760.91	–	–

Note: Eq means Equation. RT denotes response times.

Fluency Model 2 Result

Estimated item parameters from fluency model 2

Item	q Vector $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$	Response model			Response time	
		g	s_1	s_2	a	γ
Item1	(1000)	0.802	0.029	0.010	1.661	3.193
Item2	(0100)	0.771	0.039	0.013	1.803	3.042
Item3	(0001)	0.488	0.160	0.041	1.861	3.559
Item4	(0010)	0.586	0.023	0.010	1.955	3.357
Item5	(1100)	0.503	0.071	0.030	1.642	3.539
Item6	(0110)	0.209	0.453	0.264	1.523	3.788
Item7	(1100)	0.660	0.048	0.021	1.626	3.366
Item8	(1001)	0.396	0.103	0.050	1.555	3.610
Item9	(0010)	0.827	0.010	0.004	2.116	2.789
Item10	(0110)	0.583	0.082	0.041	1.464	3.445

The order of the attributes in a q vector is $\alpha_1 = x90$, $\alpha_2 = y90$, $\alpha_3 = x180$ and $\alpha_4 = y180$.

Fluency Model 2 Result

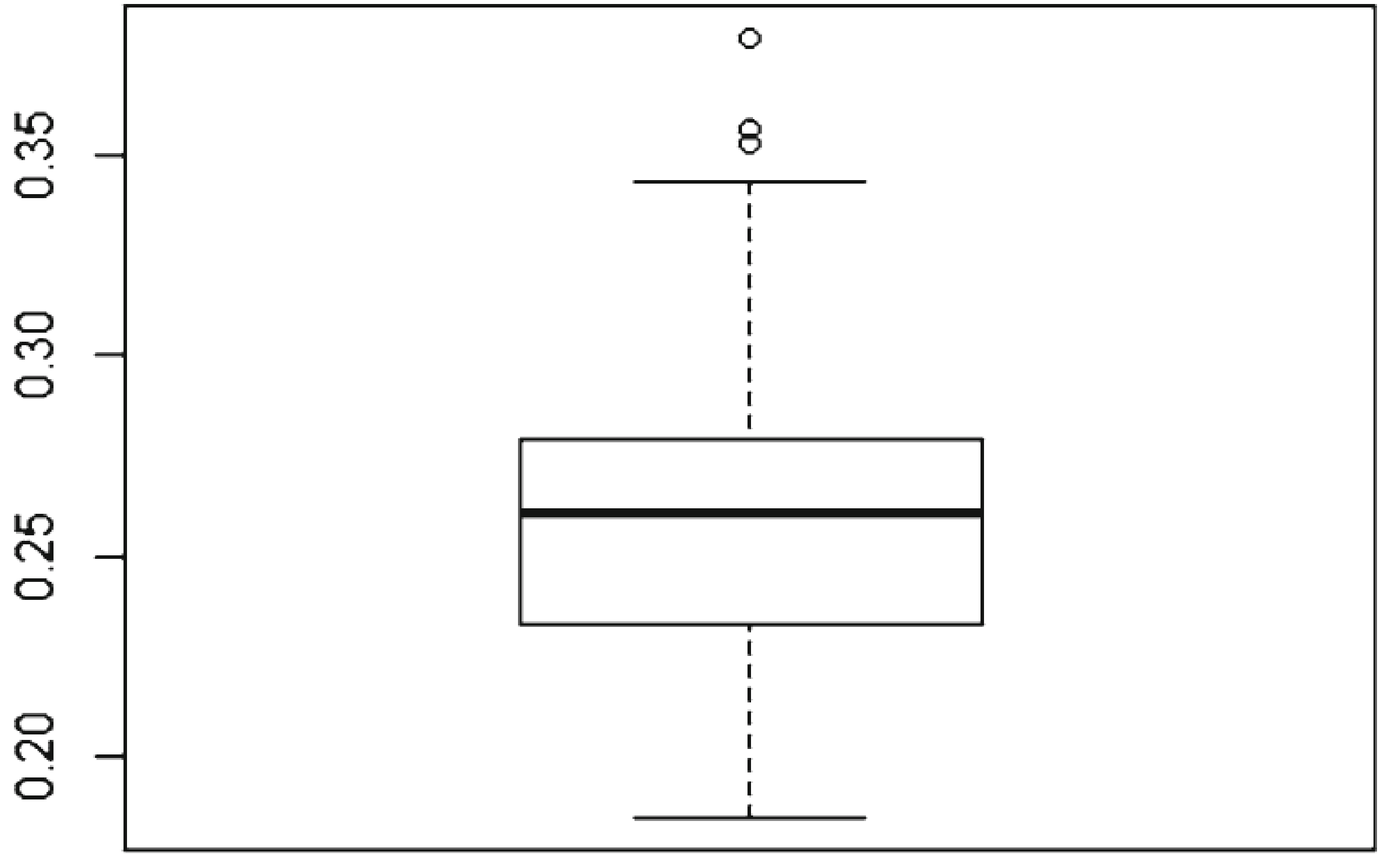


FIGURE 4.
Distribution of ϕ_i

Is the Proposed Fluency Model More Informative?

Summary information for three groups

Group	Mean attribute level				Sample size
	α_1	α_2	α_3	α_4	
Group1	1.012	1.383	1.086	1.753	81
Group2	1.000	1.000	1.000	1.000	122
Group3	0.722	0.583	0.806	0.722	36

Note: $\alpha_1 = x90$, $\alpha_2 = y90$, $\alpha_3 = x180$ and $\alpha_4 = y180$.

Is the Proposed Fluency Model More Informative?

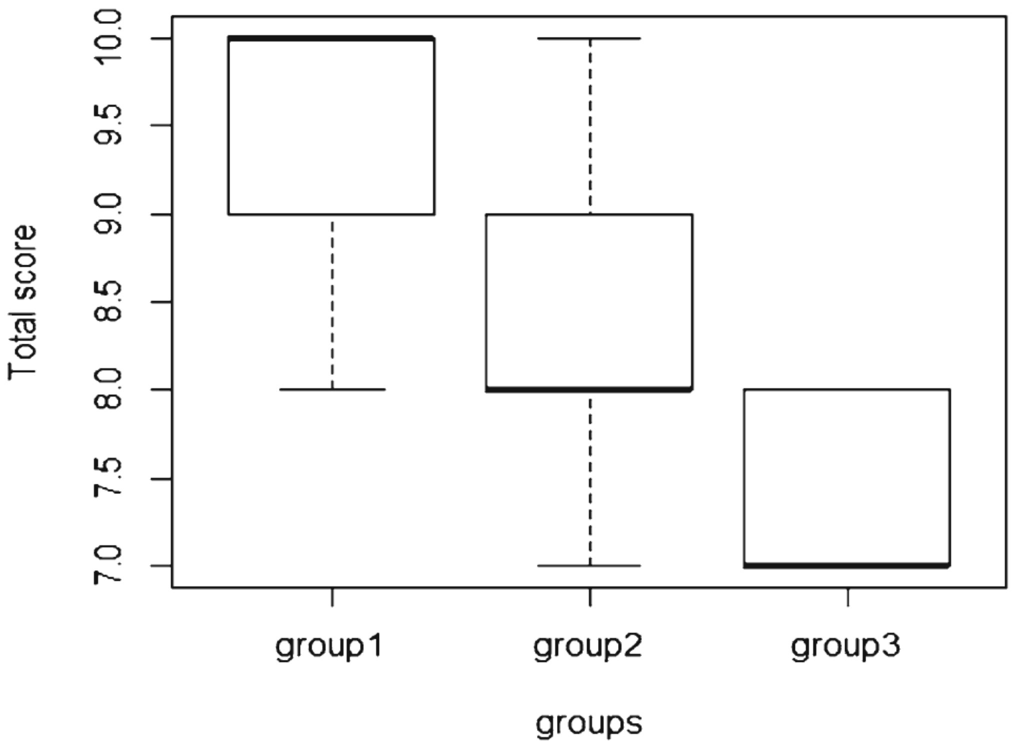


FIGURE 5.
Boxplot of total scores for three groups

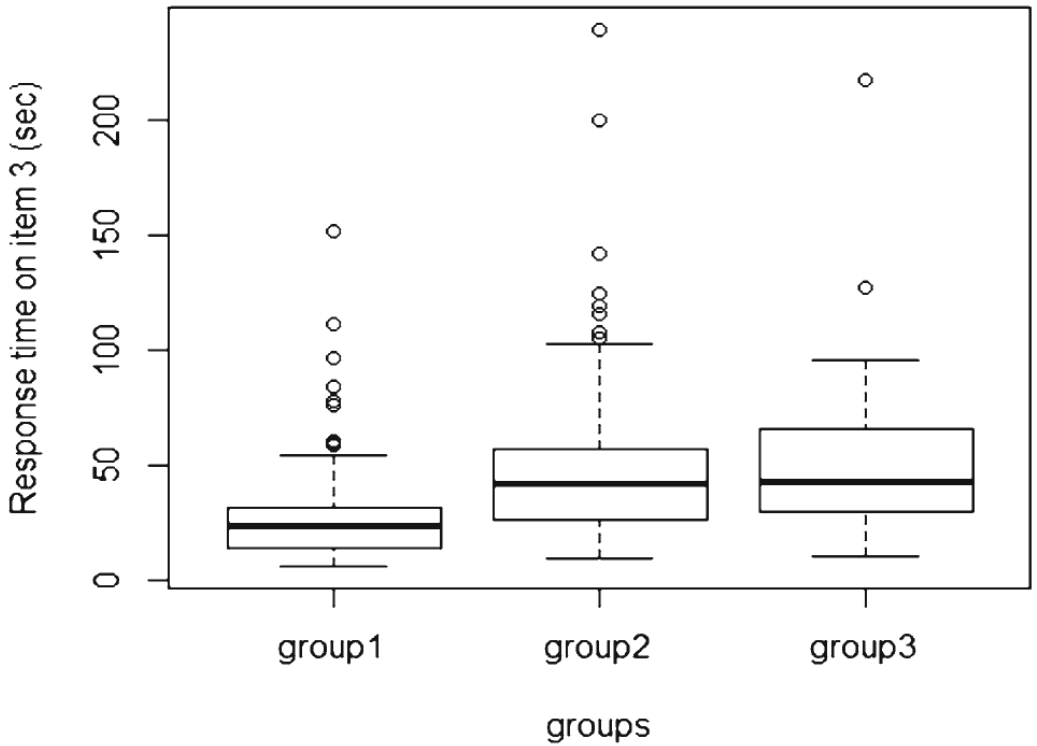


FIGURE 6.
Boxplot of response time on item 3

Simulations Based on Real Data Conditions

- How information from correct response times can improve the classification accuracy?
  - the true model parameters: the estimated values from the real data application
  - data generation: 351 students' responses to 10 questions
    1. true model: model 1 + model 2 (covariate function as $1_{\{\eta_{ij}=2\}}$)
    2. true model: model 1
  - evaluation criteria: the attribute-wise agreement rate (AAR)

The Attribute-wise Classification Accuracy

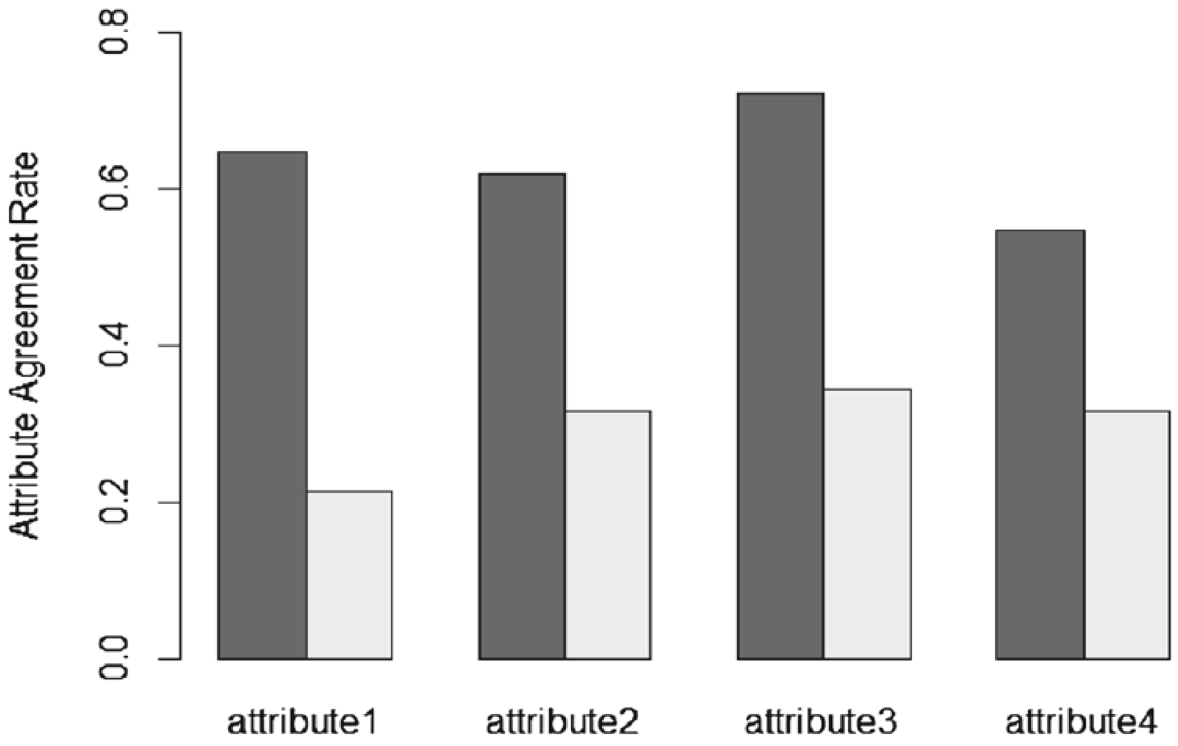


FIGURE 7.
Comparison of attribute agreement rate for two models. The grey bar represents the AAR from the CDM fluency model 2; the white bar represents the AAR from the baseline model 2

Simulation Studies

- Simulation conditions:
  - the sample size (N): 500, 1000
  - the test length (J): 20, 40
  - the true distribution for ϕ_i:

    $N(1, 0.6)1_{\{\phi_i > 1\}}$ → 2.7s – 4.5s

    $N(0.6, 0.3)1_{\{\phi_i > 0.4\}}$ → 1.5s – 2.0s
  - the item parameters for the response model (g, s_1, s_2):

    small measurement errors → (0.1, 0.2, 0.1)

    moderate measurement errors → (0.1, 0.5, 0.1)
  - the data generation model

Fluency	Response	RT	$g(\boldsymbol{\alpha}_i, \mathbf{q}_j)$
1	Eq. (1)	Eq. (2)	η_{ij}
2	Eq. (1)	Eq. (2)	$1_{\{\eta_{ij}=2\}}$
3	Eq. (1)	Eq. (3)	η_{ij}
4	Eq. (1)	Eq. (3)	$1_{\{\eta_{ij}=2\}}$
5	Eq. (1)	–	–

Simulation Studies

- Simulation conditions:
  - the number of attributes: 4 (possible states 3^4)
  - the response time model parameters:

    a from N(3.5, 0.5)

    γ from uniform (2,4)
  - the speed parameter:

    for models 1 & 2: $\tau_i \sim N(0, 0.5)$

    for models 3 & 4: $(\tau_{0i}, \tau_{1i}) \sim \mathrm{MVN}((0,0), \Sigma_{\tau_0\tau_1})$ $\Sigma_{\tau_0\tau_1} = \begin{pmatrix} 0.305 & 0.122 \\ 0.122 & 0.15 \end{pmatrix}$

- Evaluation criteria:
  - the MCMC chain convergence: the maximum PSRF
  - the parameter recovery: AAR & PAR & correlation & deviation (median, proportion)

Model Convergence Results

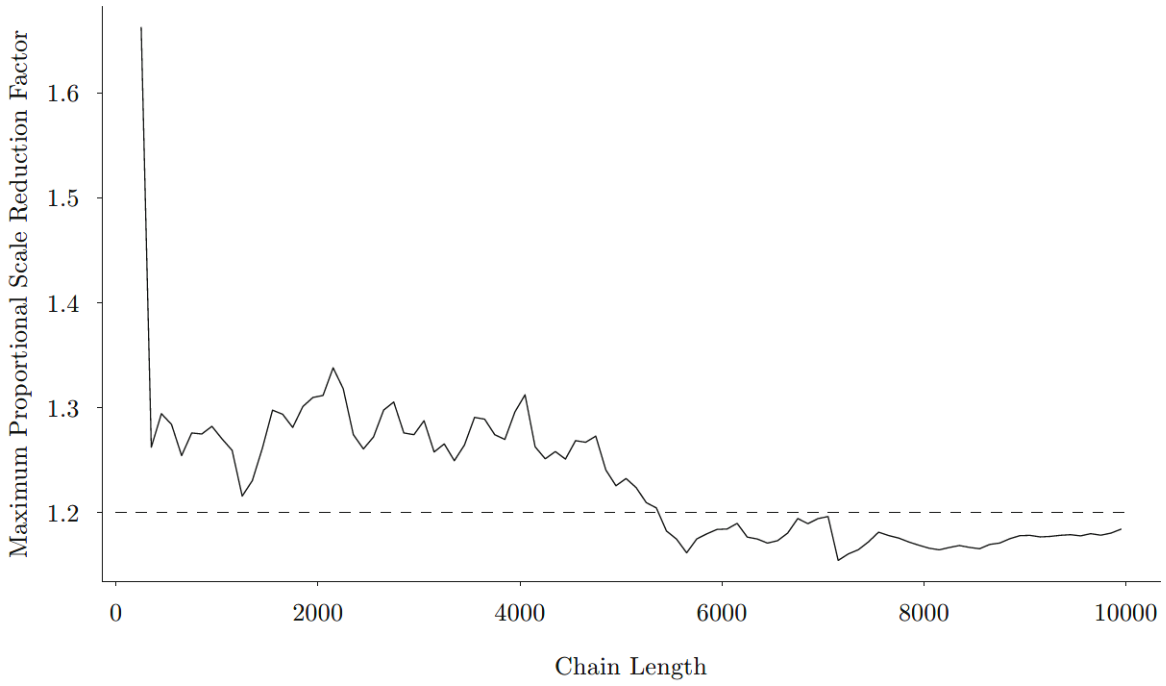


FIGURE 8.
$\hat{R}$ plot of model 1

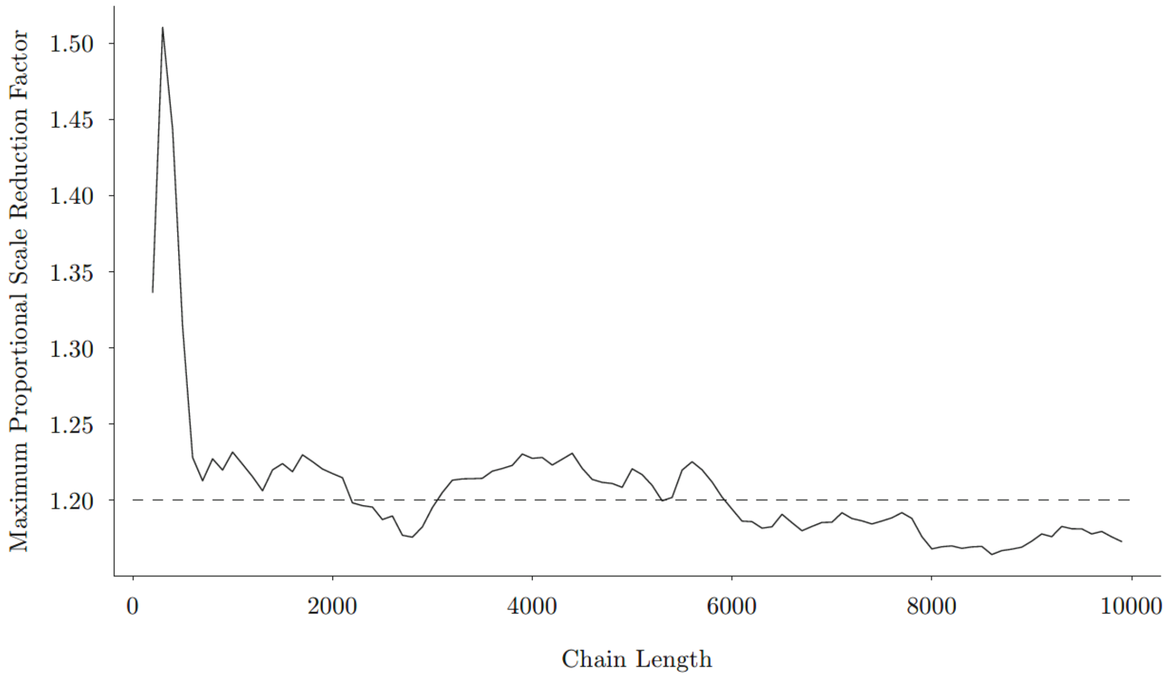


FIGURE 9.
$\hat{R}$ plot of model 3

Parameter Recovery and Model Properties

$N(1, 0.6)1_{\{\phi_i > 1\}}$

Classification results of ϕ generation condition 1

Cond.	Model	J	N	PAR	AAR1	AAR2	AAR3	AAR4	ϕ cor	Median	Pr
1	1	20	500	0.858 (0.008)	0.912	0.919	0.927	0.928	0.619 (0.025)	0.059	0.816
		40	1000	0.894 (0.011)	0.947	0.950	0.941	0.946	0.689 (0.023)	0.045	0.857
	2	20	500	0.829 (0.013)	0.948	0.945	0.925	0.944	0.715 (0.034)	0.077	0.880
(g, s_1, s_2)		40	1000	0.896 (0.012)	0.964	0.969	0.956	0.965	0.775 (0.035)	0.067	0.908
$(0.1, 0.2, 0.1)$	3	20	500	0.839 (0.012)	0.944	0.944	0.921	0.949	0.645 (0.016)	0.093	0.742
		40	1000	0.883 (0.010)	0.943	0.950	0.941	0.946	0.829 (0.009)	0.096	0.807
	4	20	500	0.780 (0.012)	0.924	0.933	0.913	0.933	0.641 (0.031)	0.080	0.762
		40	1000	0.844 (0.011)	0.949	0.952	0.940	0.946	0.865 (0.021)	0.083	0.862
2	1	20	500	0.845 (0.008)	0.931	0.938	0.910	0.943	0.694 (0.024)	0.052	0.856
		40	1000	0.925 (0.007)	0.967	0.970	0.961	0.965	0.755 (0.021)	0.040	0.911
	2	20	500	0.589 (0.014)	0.875	0.878	0.817	0.880	0.682 (0.036)	0.078	0.887
(g, s_1, s_2)		40	1000	0.790 (0.013)	0.937	0.945	0.917	0.931	0.746 (0.041)	0.067	0.900
$(0.1, 0.5, 0.1)$	3	20	500	0.815 (0.012)	0.895	0.924	0.911	0.922	0.627 (0.018)	0.092	0.777
		40	1000	0.815 (0.015)	0.900	0.911	0.901	0.907	0.819 (0.015)	0.102	0.781
	4	20	500	0.610 (0.014)	0.885	0.883	0.848	0.890	0.660 (0.035)	0.085	0.783
		40	1000	0.781 (0.013)	0.942	0.946	0.917	0.933	0.861 (0.032)	0.087	0.848

Parameter Recovery and Model Properties

Cond.	Model	J	N	$N(1, 0.6)1_{\{\phi_i>1\}}$ ϕ cor	Median	Pr	$N(0.6, 0.3)1_{\{\phi_i>0.4\}}$ ϕ cor	Median	Pr
1	1	20	500	0.619 (0.025)	0.059	0.816	0.667 (0.023)	0.154	0.614
		40	1000	0.689 (0.023)	0.045	0.857	0.680 (0.015)	0.120	0.650
	2	20	500	0.715 (0.034)	0.077	0.880	0.721 (0.033)	0.158	0.567
(g, s_1, s_2)		40	1000	0.775 (0.035)	0.067	0.908	0.767 (0.048)	0.146	0.667
$(0.1, 0.2, 0.1)$	3	20	500	0.645 (0.016)	0.093	0.742	0.649 (0.027)	0.184	0.530
		40	1000	0.829 (0.009)	0.096	0.807	0.755 (0.015)	0.143	0.634
	4	20	500	0.641 (0.031)	0.080	0.762	0.668 (0.045)	0.202	0.494
		40	1000	0.865 (0.021)	0.083	0.862	0.745 (0.035)	0.170	0.570
2	1	20	500	0.694 (0.024)	0.052	0.856	0.745 (0.024)	0.127	0.642
		40	1000	0.755 (0.021)	0.040	0.911	0.695 (0.020)	0.105	0.629
	2	20	500	0.682 (0.036)	0.078	0.887	0.715 (0.052)	0.159	0.541
(g, s_1, s_2)		40	1000	0.746 (0.041)	0.067	0.900	0.765 (0.050)	0.147	0.663
$(0.1, 0.5, 0.1)$	3	20	500	0.627 (0.018)	0.092	0.777	0.664 (0.028)	0.187	0.530
		40	1000	0.819 (0.015)	0.102	0.781	0.716 (0.016)	0.153	0.597
	4	20	500	0.660 (0.035)	0.085	0.783	0.645 (0.051)	0.206	0.488
		40	1000	0.861 (0.032)	0.087	0.848	0.741 (0.038)	0.170	0.568

Parameter Recovery and Model Properties

Results of τ_i estimation of model 1 and 2

ϕ Cond.	Item Cond.	Model	J	N	τ_i cor	Median	Pr
1	1	1	20	500	0.968 (0.005)	0.171	0.598
			40	1000	0.982 (0.004)	0.142	0.592
		2	20	500	0.953 (0.009)	0.174	0.562
			40	1000	0.982 (0.005)	0.131	0.611
	2	1	20	500	0.978 (0.003)	0.157	0.573
			40	1000	0.989 (0.003)	0.130	0.629
		2	20	500	0.953 (0.009)	0.176	0.550
			40	1000	0.983 (0.004)	0.132	0.598
2	1	1	20	500	0.974 (0.005)	0.184	0.521
			40	1000	0.988 (0.008)	0.141	0.627
		2	20	500	0.970 (0.003)	0.168	0.564
			40	1000	0.983 (0.004)	0.133	0.641
	2	1	20	500	0.980 (0.005)	0.155	0.561
			40	1000	0.981 (0.006)	0.121	0.630
		2	20	500	0.973 (0.003)	0.166	0.540
			40	1000	0.984 (0.004)	0.146	0.603

Parameter Recovery and Model Properties

Results of τ_{0i} and τ_{1i} estimation of model 3 and 4

ϕ Cond.	Item Cond.	Model	J	N	τ_0 cor	Median	Pr	τ_1 cor	Median	Pr
1	1	3	20	500	0.975 (0.003)	0.253	0.424	0.622 (0.014)	1.481	0.177
			40	1000	0.987 (0.001)	0.205	0.494	0.688 (0.009)	1.709	0.230
		4	20	500	0.975 (0.004)	0.191	0.516	0.868 (0.010)	1.586	0.284
			40	1000	0.988 (0.001)	0.136	0.618	0.927 (0.006)	1.449	0.370
	2	3	20	500	0.984 (0.003)	0.251	0.420	0.582 (0.013)	1.230	0.154
			40	1000	0.991 (0.001)	0.231	0.459	0.638 (0.011)	1.247	0.203
		4	20	500	0.985 (0.001)	0.175	0.543	0.871 (0.013)	1.797	0.264
			40	1000	0.992 (0.001)	0.126	0.644	0.928 (0.012)	1.496	0.347
2	1	3	20	500	0.975 (0.003)	0.248	0.429	0.779 (0.008)	1.277	0.177
			40	1000	0.988 (0.001)	0.167	0.558	0.862 (0.009)	1.315	0.246
		4	20	500	0.975 (0.004)	0.203	0.497	0.888 (0.007)	1.375	0.276
			40	1000	0.988 (0.001)	0.142	0.607	0.925 (0.006)	1.342	0.358
	2	3	20	500	0.984 (0.002)	0.256	0.416	0.737 (0.008)	1.250	0.142
			40	1000	0.991 (0.001)	0.157	0.582	0.853 (0.011)	1.253	0.226
		4	20	500	0.984 (0.002)	0.185	0.524	0.894 (0.010)	1.430	0.266
			40	1000	0.991 (0.001)	0.131	0.639	0.934 (0.008)	1.349	0.350

Parameter F

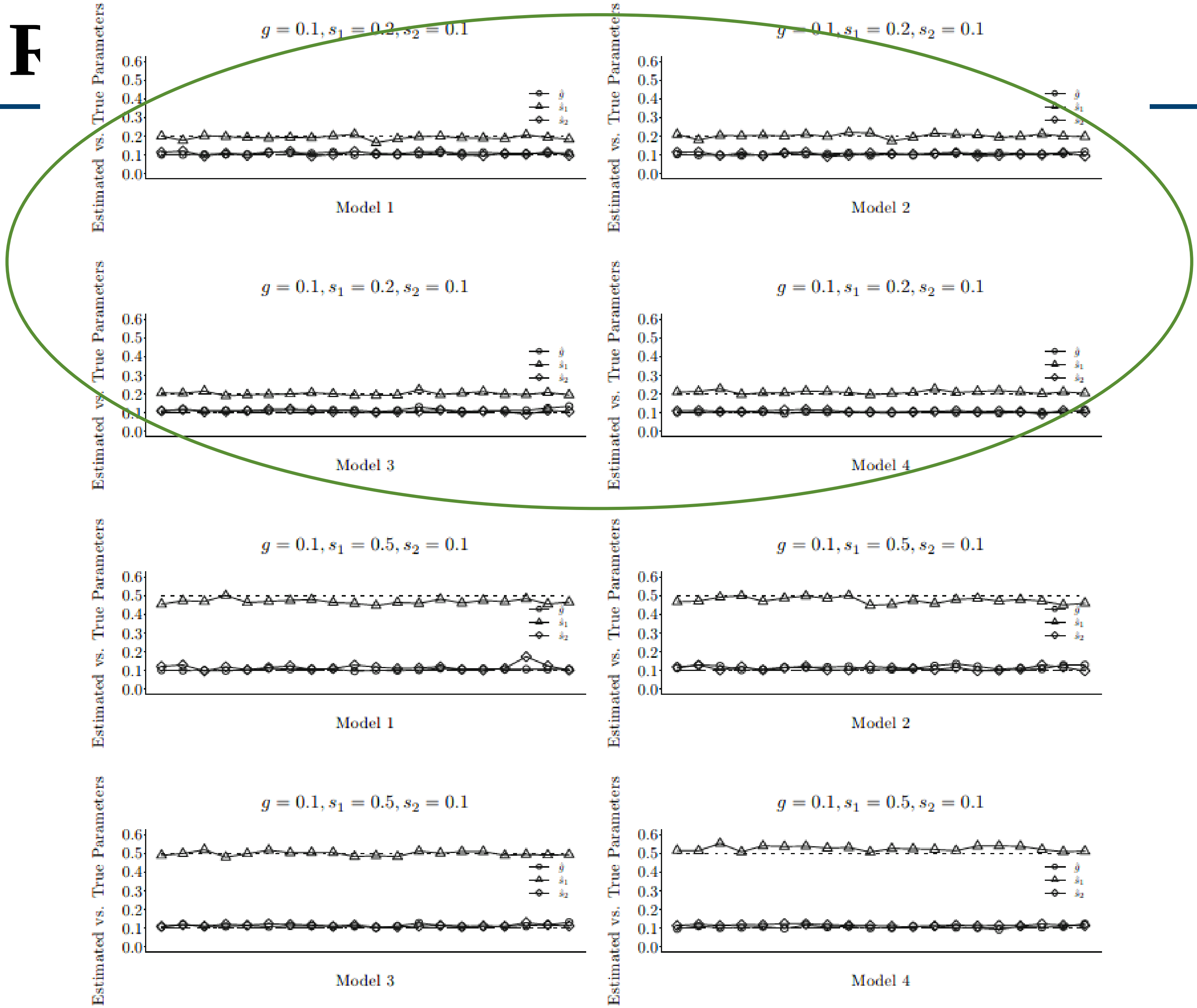


FIGURE 10.
Response model item parameter estimation for $N = 500$, $J = 20$ and ϕ condition 1. The true values are denoted by dash lines

Parameter E

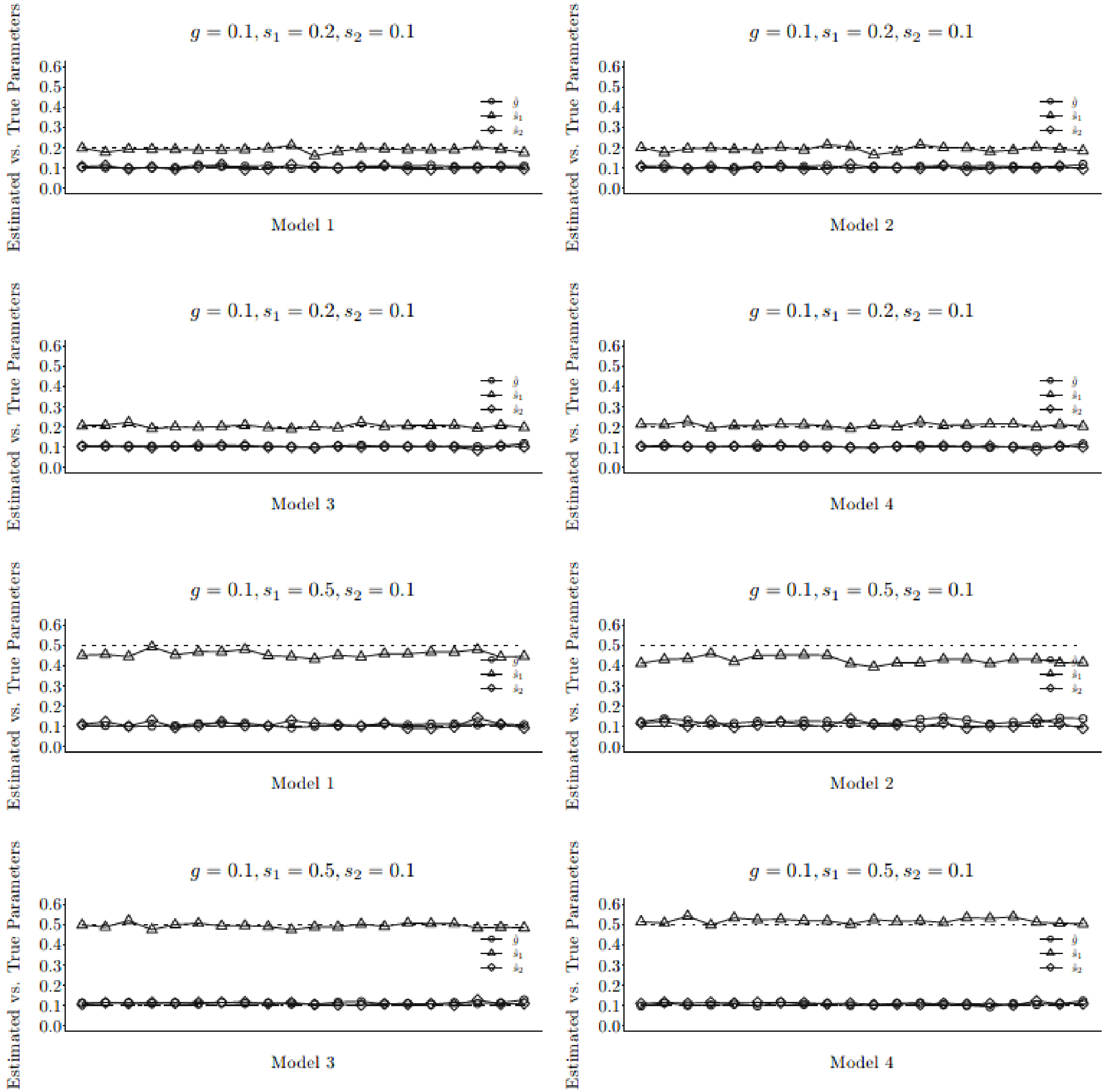


FIGURE 11.
Response model item parameter estimation for $N = 500$, $J = 20$ and ϕ condition 2. The true values are denoted by dash lines

Discussion

- this study offers a new view to measure skill accuracy and fluency
  - enable teachers to instruct students to work on reinforcing the accurate albeit not yet fluent skills
- the proposed joint models were able to reveal more information regarding test takers' spatial skills
- further improvement:
  - considering response heterogeneity (mixture model or nonlinear assumptions)
  - investigating the issue of local dependencies between response accuracy and time within items
  - relaxing some restrict assumptions

THANKS FOR LISTENING!

REPORTER

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