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A psychometric model for respondent-level anchoring on self-report rating scale instruments

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Introduction

- The self-report rating scale format
    - low-stakes settings: a lack of respondent effort
    - careless responses are less clear

What can be an indicator of insufficient respondent effort?

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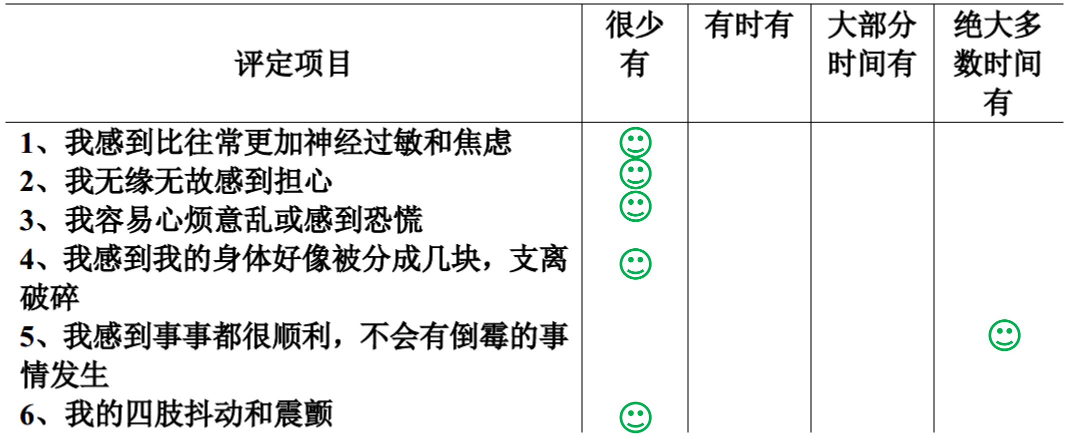
What can be an indicator of insufficient respondent effort?



评定项目	很少有	有时有	大部分时间有	绝大多数时间有
1、我感到比往常更加神经过敏和焦虑	☺			
2、我无缘无故感到担心	☺			
3、我容易心烦意乱或感到恐慌	☺			
4、我感到我的身体好像被分成几块，支离破碎	☺			
5、我感到事事都很顺利，不会有倒霉的事情发生				☺
6、我的四肢抖动和震颤	☺			

Self-Rating Anxiety Scale, SAS (Zung, 1971)

same content (anchor):
select the same or a nearby response

different content / reversed direction (without anchor):
go away

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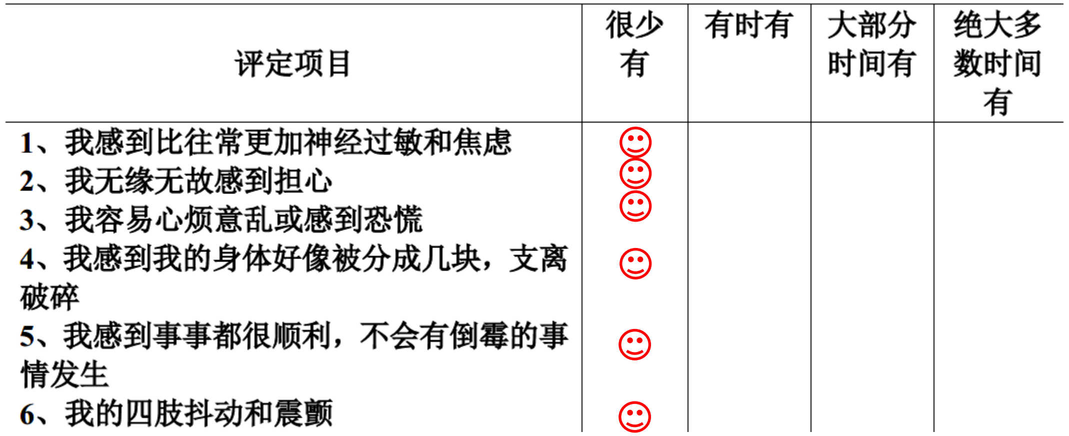

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Self-Rating Anxiety Scale, SAS (Zung, 1971)

same content (anchor):
select the same or a nearby response

different content / reversed direction (with anchor):
lack of a justification

Introduction

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  - careless responses are less clear

What can be an indicator of insufficient respondent effort?

- The solutions
  - measuring anchoring behaviour
  - consider response styles
  - explore the potential for a shared underlying commonality

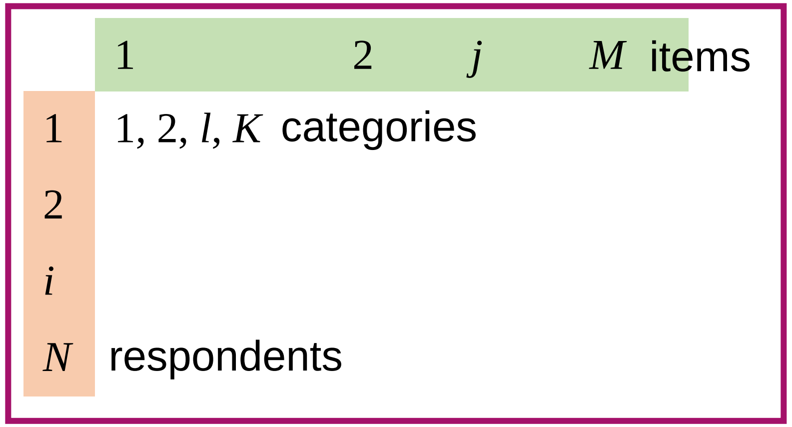
Model

- an extension of a multidimensional nominal model

$$\mathbb{P}\left(X_{ij}=k\right)=\frac{\exp\left(q_{ijk}\right)}{\sum_{l=1}^{K}\exp\left(q_{ijl}\right)}$$

  - latent preference for category k

$$q_{ijk}=\begin{cases} a_{jk}\theta_i+c_{jk}+\eta_{ik}, & j=1, \\ a_{jk}\theta_i+c_{jk}+\eta_{ik}+\rho_i\lambda_{\left|X_{i,\ j-1}-k\right|}, & j\geq 2 \end{cases}$$



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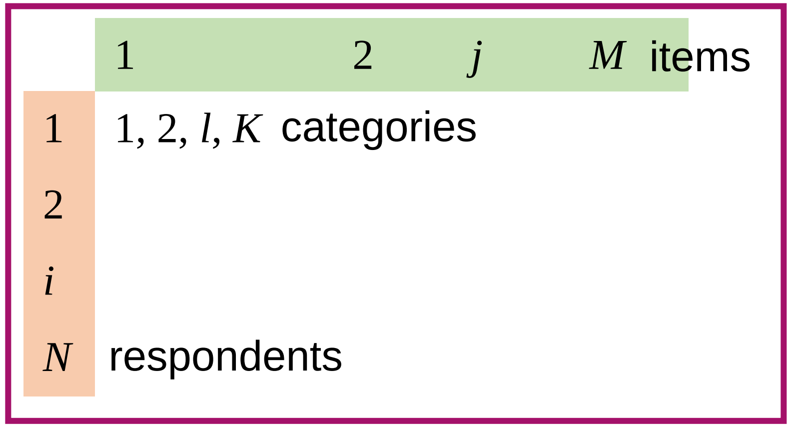


1 2 j M items

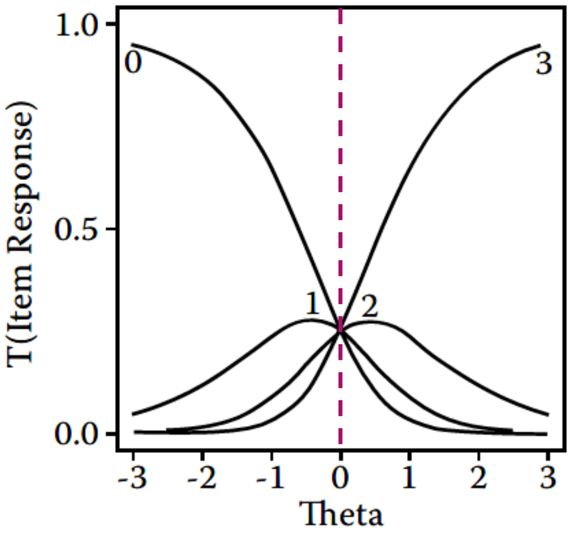
1 $1, 2, l, K$ categories

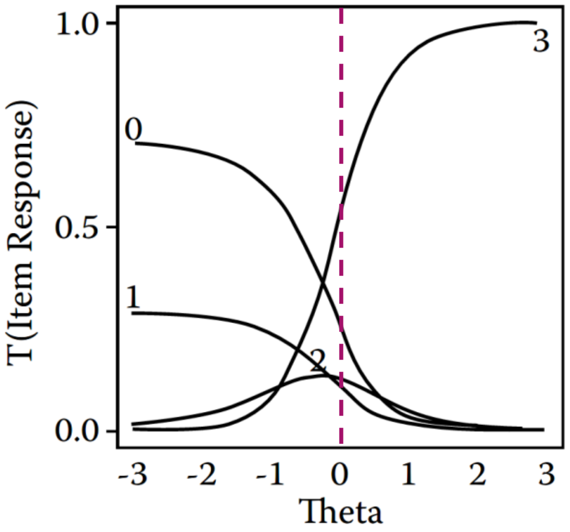
2

i

N respondents

Response Category (k)	Item 1		Item 2	
	a	c	a	c
dog 0	0.0	0.0	0.0	0.0
cat 1	1.0	0.0	0.0	–0.9
bird 2	2.0	0.0	1.1	–0.7
bear 3	3.0	0.0	2.7	0.7





David Thissen, Li Cai, R. Darrell Bock, Book Chapter

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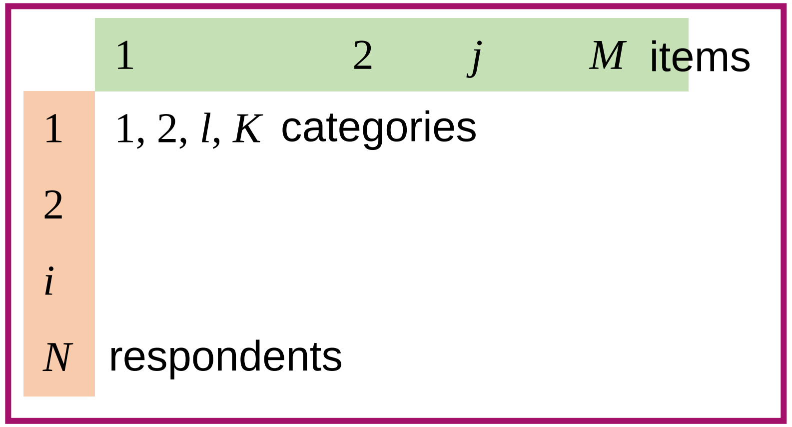
$$q_{ijk}=\begin{cases} a_{jk}\theta_i+c_{jk}+\eta_{ik}, & j=1,\\ a_{jk}\theta_i+c_{jk}+\eta_{ik}+\rho_i\lambda_{\boxed{|X_{i,\ j-1}-k|}}, & j\geq 2 \end{cases}$$

the strength of this anchoring effect for respondent i $(\rho_i>0)$
the larger, the more likely to reduce effort

distance from the category previous selected: $X_{i,\ j-1}-k$
potential anchoring effect: $\lambda_0,\lambda_1,\cdots,\lambda_{K-1}$ $(\lambda_0>\lambda_1>\lambda_2>\cdots>\lambda_{K-1})$



	1	2	j	M items
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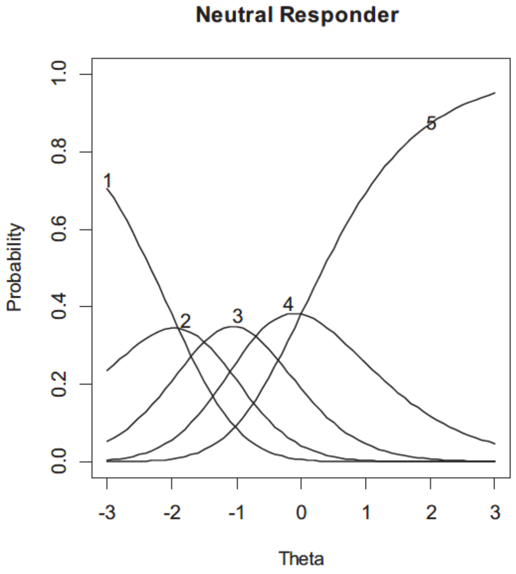
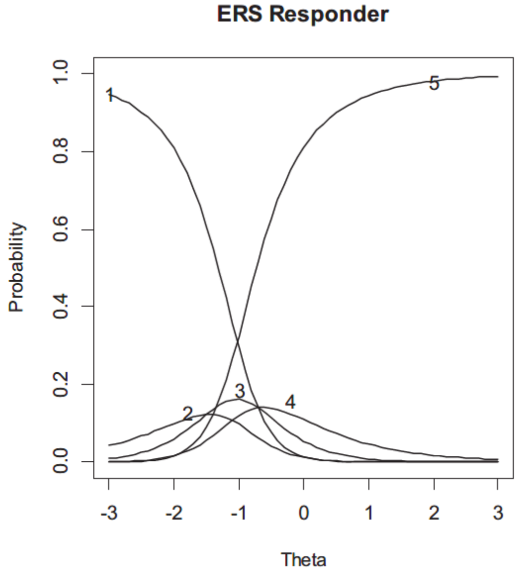
choose category k for reasons unrelated to θ_i (response style)

[0, 0, 0, 0, 0]



[2, -1.3, -1.3, -1.3, 2]



Bolt et al., 2014, Psychological Methods

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  - identification constraints

$$\sum_{k=1}^{K}c_{jk} = 0 \text{ for all } j,\quad \sum_{k=1}^{K}\eta_{ik} = 0 \text{ for all } i,\quad \prod_{i=1}^{N}\rho_i = 1,\ \text{ and } \sum_{d=0}^{K-1}\lambda_d = 0$$

  - equal interval scoring for each item ($\overline{a}_j>0$)

$$a_{jk} = k\cdot\overline{a}_j$$

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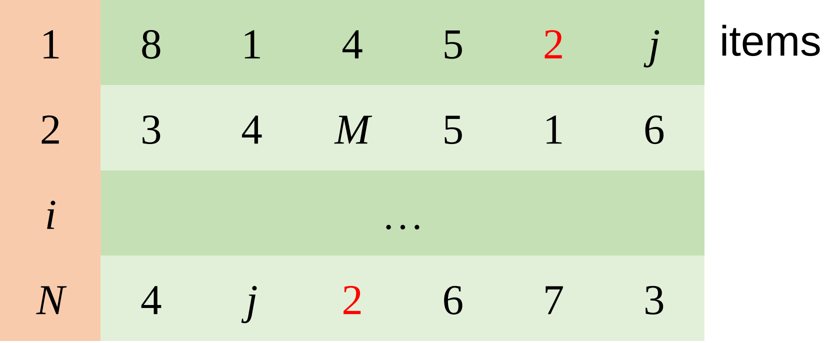
reversed items are in red



1	8	1	4	5	2	j
2	3	4	M	5	1	6
i			…			
N	4	j	2	6	7	3

respondents

How to represent items that are reverse oriented?

What if the items are presented in random order?

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items

respondents

How to represent items that are reverse oriented?

→ latent trait level increase, preference decrease

What if the items are presented in random order?

→ the item index does not equal to the location

Model

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$$\mathbb{P}(X_{ij} = k) = \frac{\exp\left(q_{ijk}\right)}{\sum_{l=1}^{K} \exp\left(q_{ijl}\right)}$$

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$$q_{ijk} = \begin{cases} a_{jk}\theta_i + c_{jk} + \eta_{ik}, & j = 1, \\ a_{jk}\theta_i + c_{jk} + \eta_{ik} + \rho_i \lambda_{\left|X_{i,\ j-1} - k\right|}, & j \geq 2 \end{cases}$$

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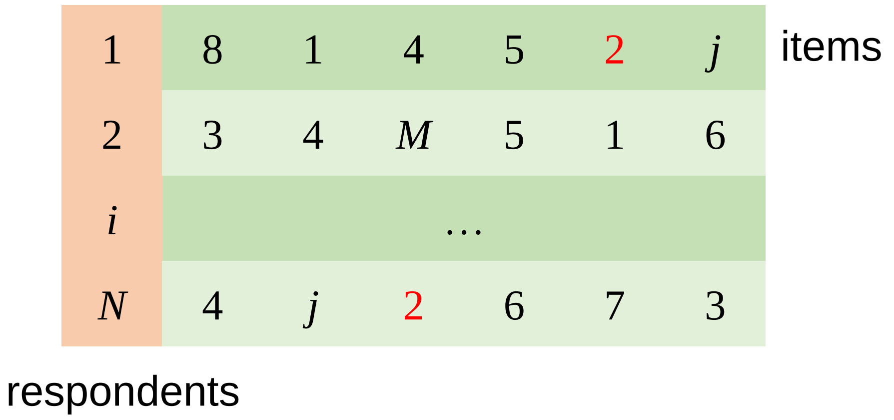

1	8	1	4	5	2	j
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  - **Step 1:** identify the previous item l_{ij} (the item index of the previous one item; the first item $l_{ij} = 0$)
  - **Step 2:** define the direction $s_j = \begin{cases} 1, & \text{if item } j \text{ is oriented in the same direction as item } 1, \\ -1, & \text{if item } j \text{ is oriented in the opposite direction to item } 1 \end{cases}$

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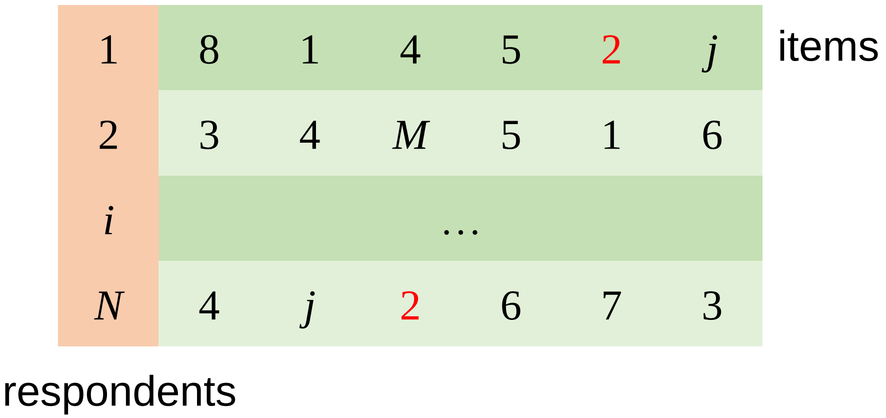

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  - **Step 3:** adjust the item directionality

$$q_{ijk}=\begin{cases} \left(s_j a_{jk}\right)\theta_i+c_{jk}+\eta_{ik}, & \text{if } l_{ij}=0 \text{ or } s_j\neq s_{l_{ij}}, \\ \left(s_j a_{jk}\right)\theta_i+c_{jk}+\eta_{ik}+\rho_i\lambda_{\left|X_{i,\ l_{ij}}-k\right|}, & \text{otherwise.}\end{cases}$$

different direction

same direction

Bayesian estimation

- Stan (Hamiltonian Monte Carlo algorithm)
  - the priors:

$$
\begin{gathered}
\theta_i \sim \mathcal{N}(0,\,1),\;\; i = 1,\,2,\,\ldots,\,N,\\
\lambda_d \sim \mathcal{N}(0,\,1),\;\; d = 0,\,1,\,\ldots,\,K-1,\\
[\eta_{i1},\,\eta_{i2},\,\ldots,\,\eta_{iK},\,\log\rho_i]^{\mathrm{T}} \sim \mathcal{N}_{K+1}(0,\,\Sigma_\eta),\;\; i = 1,\,2,\,\ldots,\,N,\\
[c_{j1},\,c_{j2},\,\ldots,\,c_{jK},\,\log\overline{a}_j]^{\mathrm{T}} \sim \mathcal{N}_{K+1}(0,\,\Sigma_c),\;\; j = 1,\,2,\,\ldots,\,M,\\
\Sigma_c = \tau_c^{\mathrm{T}}\Omega_c\tau_c,\\
\Sigma_\eta = \tau_\eta^{\mathrm{T}}\Omega_\eta\tau_\eta,\\
\tau_c,\,\tau_\eta \sim \mathrm{HalfCauchy}(1),\\
\Omega_c,\,\Omega_\eta \sim \mathrm{LKJCorr}(2),
\end{gathered}
$$

  - convergence:

1. the Gelman–Rubin $\hat{R}$ (close to 1)
2. the ratio of the effective sample size N_{eff} to the total sample size N

```
data {                                   // Data block
int<lower = 1> N;                        // number of respondents
}

parameters {                             // Parameters block
vector[N] theta;                         // latent trait
row_vector[K] anc;                       // uncentered anchoring effects
}

transformed parameters {                 // Transformed parameters block
vector[K] p[N, M];                       // latent preferences
p[n, m, k] = u + rho[n] * lambda[1 + abs(y[n, z[n, w - 1]] - k)];
}

model {                                  // Model block
// priors
anc ~ normal(0, 1);

// target distribution
y[n, m] ~ categorical_logit(p[n, m]);
}

generated quantities {                   // Generated quantities block
omega_m = tcrossprod(L_omega_m); // correlation matrix of
item-level parameters}
```

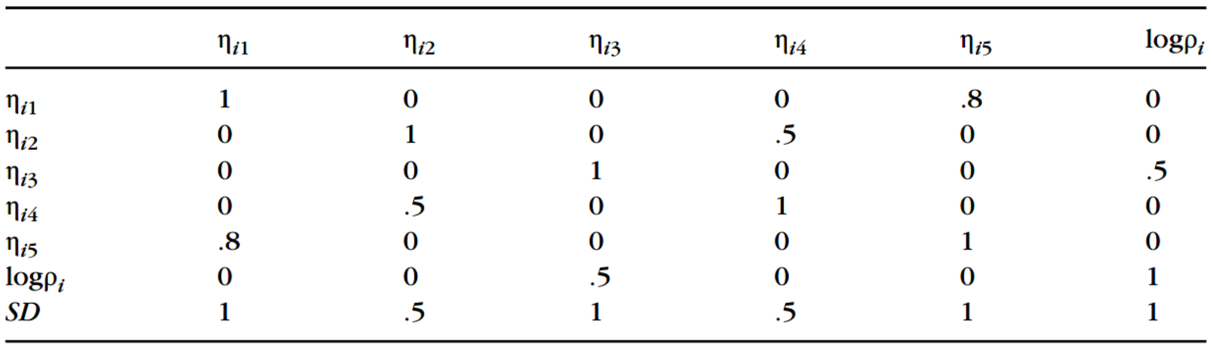
Just an example

Simulation

- Purpose: examine the parameter recovery
  - the number of items: M = 30, 50 or 100
  - the anchoring effects: $\lambda = [0.2, 0.1, 0, -0.1, -0.2]^T$ or $\lambda = [0,0,0,0,0]^T$(no anchor) Any false positive?
  - the content trait distribution: $\theta \sim N(0,1)$ or $\theta \sim N(1,1)$

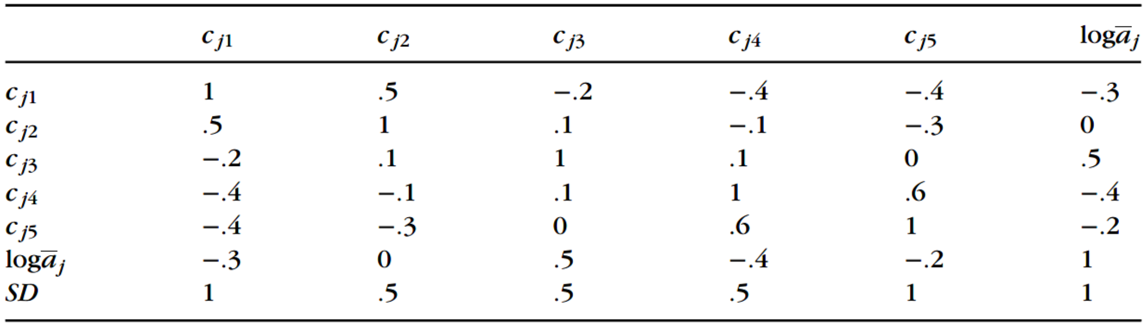
  - response categories per item: K = 5
  - item parameters: $s_j = 1$ & $[\eta_{i1}, \eta_{i2}, \ldots, \eta_{iK}, \log\rho_i]^{\mathrm{T}} \sim \mathcal{N}_{K+1}(0, \Sigma_\eta),\ i = 1, 2, \ldots, N,$
  - examinees: N = 200 $[c_{j1}, c_{j2}, \ldots, c_{jK}, \log\bar{a}_j]^{\mathrm{T}} \sim \mathcal{N}_{K+1}(0, \Sigma_c),\ i = 1, 2, \ldots, M,$

Table 1. Correlation matrix and standard deviations of respondent parameters Σ_η



	η_{i1}	η_{i2}	η_{i3}	η_{i4}	η_{i5}	$\log\rho_i$
η_{i1}	1	0	0	0	.8	0
η_{i2}	0	1	0	.5	0	0
η_{i3}	0	0	1	0	0	.5
η_{i4}	0	.5	0	1	0	0
η_{i5}	.8	0	0	0	1	0
$\log\rho_i$	0	0	.5	0	0	1
SD	1	.5	1	.5	1	1

Table 2. Correlation matrix and standard deviations of item parameters Σ_c



	c_{j1}	c_{j2}	c_{j3}	c_{j4}	c_{j5}	$\log\bar{a}_j$
c_{j1}	1	.5	−.2	−.4	−.4	−.3
c_{j2}	.5	1	.1	−.1	−.3	0
c_{j3}	−.2	.1	1	.1	0	.5
c_{j4}	−.4	−.1	.1	1	.6	−.4
c_{j5}	−.4	−.3	0	.6	1	−.2
$\log\bar{a}_j$	−.3	0	.5	−.4	−.2	1
SD	1	.5	.5	.5	1	1

Simulation

- Estimation
  - 100 replications
  - four chains (each chain 5,000 iterations)
  - burn in: the first 2,500 iterations
  - convergence: the maximum $\hat{R}$ across all parameters < 1.1
  - the posterior means observed across 10,000 iterations were used

Results

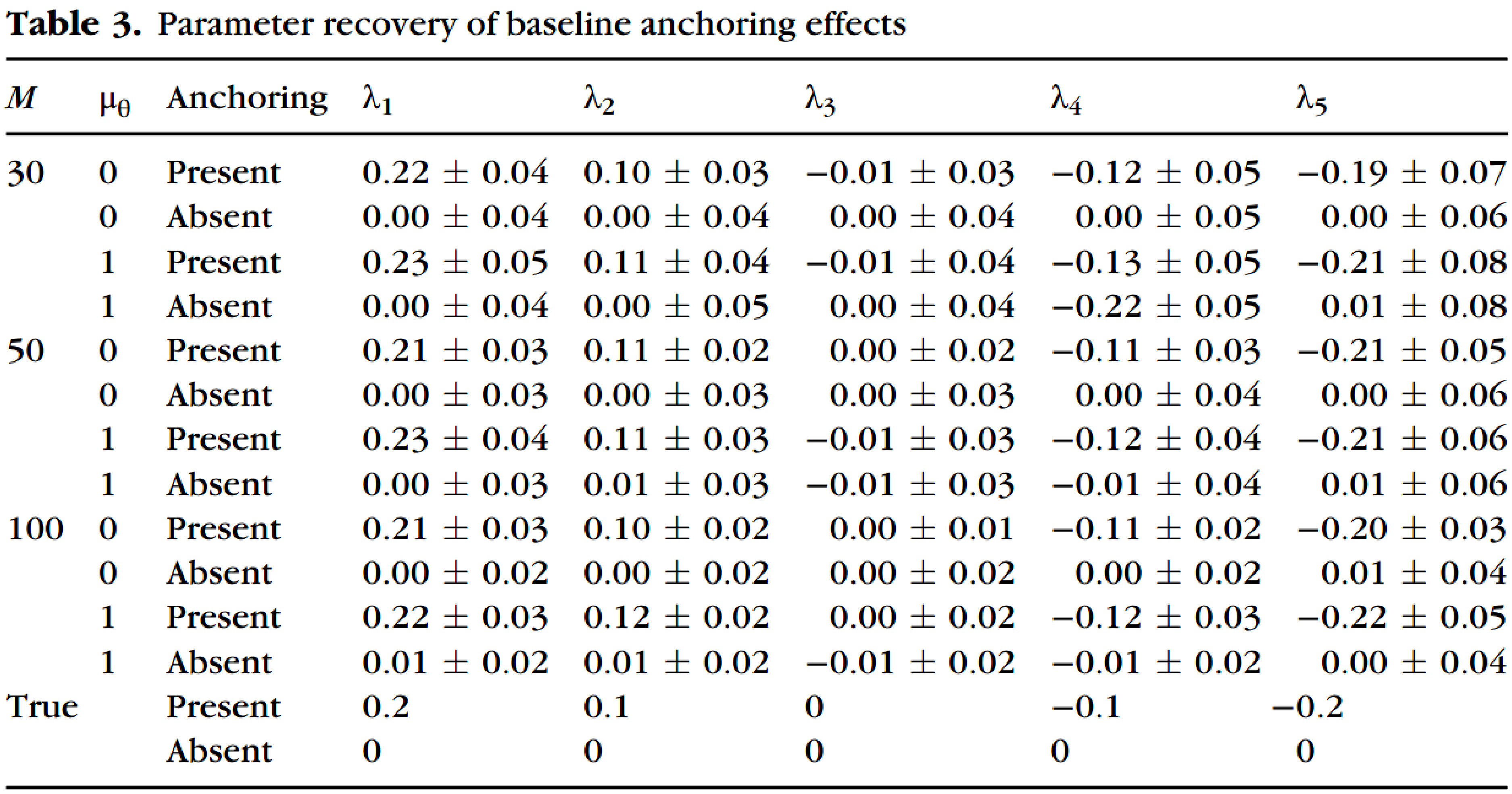


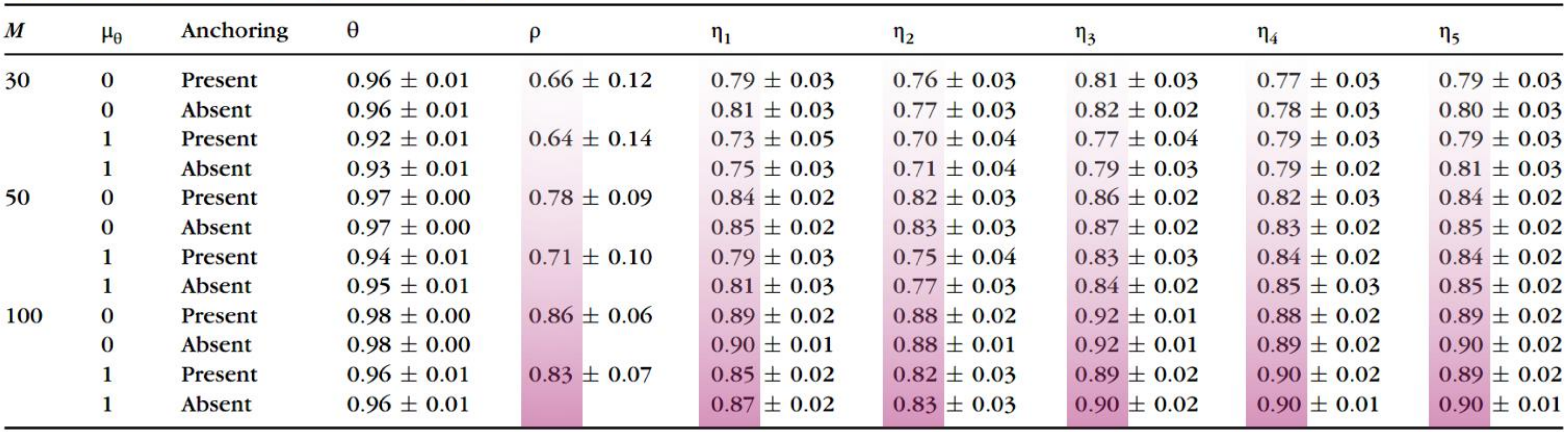
Table 3. Parameter recovery of baseline anchoring effects

M	μ_θ	Anchoring	λ_1	λ_2	λ_3	λ_4	λ_5
30	0	Present	0.22 ± 0.04	0.10 ± 0.03	−0.01 ± 0.03	−0.12 ± 0.05	−0.19 ± 0.07
	0	Absent	0.00 ± 0.04	0.00 ± 0.04	0.00 ± 0.04	0.00 ± 0.05	0.00 ± 0.06
	1	Present	0.23 ± 0.05	0.11 ± 0.04	−0.01 ± 0.04	−0.13 ± 0.05	−0.21 ± 0.08
	1	Absent	0.00 ± 0.04	0.00 ± 0.05	0.00 ± 0.04	−0.22 ± 0.05	0.01 ± 0.08
50	0	Present	0.21 ± 0.03	0.11 ± 0.02	0.00 ± 0.02	−0.11 ± 0.03	−0.21 ± 0.05
	0	Absent	0.00 ± 0.03	0.00 ± 0.03	0.00 ± 0.03	0.00 ± 0.04	0.00 ± 0.06
	1	Present	0.23 ± 0.04	0.11 ± 0.03	−0.01 ± 0.03	−0.12 ± 0.04	−0.21 ± 0.06
	1	Absent	0.00 ± 0.03	0.01 ± 0.03	−0.01 ± 0.03	−0.01 ± 0.04	0.01 ± 0.06
100	0	Present	0.21 ± 0.03	0.10 ± 0.02	0.00 ± 0.01	−0.11 ± 0.02	−0.20 ± 0.03
	0	Absent	0.00 ± 0.02	0.00 ± 0.02	0.00 ± 0.02	0.00 ± 0.02	0.01 ± 0.04
	1	Present	0.22 ± 0.03	0.12 ± 0.02	0.00 ± 0.02	−0.12 ± 0.03	−0.22 ± 0.05
	1	Absent	0.01 ± 0.02	0.01 ± 0.02	−0.01 ± 0.02	−0.01 ± 0.02	0.00 ± 0.04
True		Present	0.2	0.1	0	−0.1	−0.2
		Absent	0	0	0	0	0

the mean and standard deviation across 100 replications

Results

Table 4. Correlations of true and estimated parameters



M	μ_θ	Anchoring	θ	ρ	η_1	η_2	η_3	η_4	η_5
30	0	Present	0.96 ± 0.01	0.66 ± 0.12	0.79 ± 0.03	0.76 ± 0.03	0.81 ± 0.03	0.77 ± 0.03	0.79 ± 0.03
	0	Absent	0.96 ± 0.01		0.81 ± 0.03	0.77 ± 0.03	0.82 ± 0.02	0.78 ± 0.03	0.80 ± 0.03
	1	Present	0.92 ± 0.01	0.64 ± 0.14	0.73 ± 0.05	0.70 ± 0.04	0.77 ± 0.04	0.79 ± 0.03	0.79 ± 0.03
	1	Absent	0.93 ± 0.01		0.75 ± 0.03	0.71 ± 0.04	0.79 ± 0.03	0.79 ± 0.02	0.81 ± 0.03
50	0	Present	0.97 ± 0.00	0.78 ± 0.09	0.84 ± 0.02	0.82 ± 0.03	0.86 ± 0.02	0.82 ± 0.03	0.84 ± 0.02
	0	Absent	0.97 ± 0.00		0.85 ± 0.02	0.83 ± 0.03	0.87 ± 0.02	0.83 ± 0.02	0.85 ± 0.02
	1	Present	0.94 ± 0.01	0.71 ± 0.10	0.79 ± 0.03	0.75 ± 0.04	0.83 ± 0.03	0.84 ± 0.02	0.84 ± 0.02
	1	Absent	0.95 ± 0.01		0.81 ± 0.03	0.77 ± 0.03	0.84 ± 0.02	0.85 ± 0.03	0.85 ± 0.02
100	0	Present	0.98 ± 0.00	0.86 ± 0.06	0.89 ± 0.02	0.88 ± 0.02	0.92 ± 0.01	0.88 ± 0.02	0.89 ± 0.02
	0	Absent	0.98 ± 0.00		0.90 ± 0.01	0.88 ± 0.01	0.92 ± 0.01	0.89 ± 0.02	0.90 ± 0.02
	1	Present	0.96 ± 0.01	0.83 ± 0.07	0.85 ± 0.02	0.82 ± 0.03	0.89 ± 0.02	0.90 ± 0.02	0.89 ± 0.02
	1	Absent	0.96 ± 0.01		0.87 ± 0.02	0.83 ± 0.03	0.90 ± 0.02	0.90 ± 0.01	0.90 ± 0.01

Results

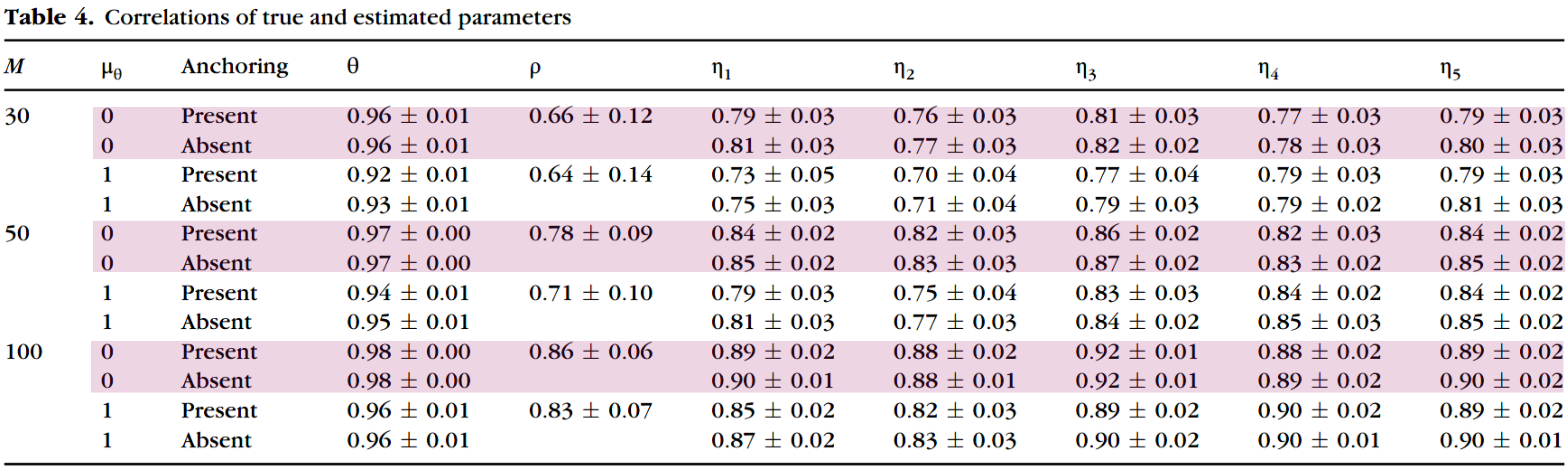


Table 4. Correlations of true and estimated parameters

M	μ_θ	Anchoring	θ	ρ	η_1	η_2	η_3	η_4	η_5
30	0	Present	0.96 ± 0.01	0.66 ± 0.12	0.79 ± 0.03	0.76 ± 0.03	0.81 ± 0.03	0.77 ± 0.03	0.79 ± 0.03
	0	Absent	0.96 ± 0.01		0.81 ± 0.03	0.77 ± 0.03	0.82 ± 0.02	0.78 ± 0.03	0.80 ± 0.03
	1	Present	0.92 ± 0.01	0.64 ± 0.14	0.73 ± 0.05	0.70 ± 0.04	0.77 ± 0.04	0.79 ± 0.03	0.79 ± 0.03
	1	Absent	0.93 ± 0.01		0.75 ± 0.03	0.71 ± 0.04	0.79 ± 0.03	0.79 ± 0.02	0.81 ± 0.03
50	0	Present	0.97 ± 0.00	0.78 ± 0.09	0.84 ± 0.02	0.82 ± 0.03	0.86 ± 0.02	0.82 ± 0.03	0.84 ± 0.02
	0	Absent	0.97 ± 0.00		0.85 ± 0.02	0.83 ± 0.03	0.87 ± 0.02	0.83 ± 0.02	0.85 ± 0.02
	1	Present	0.94 ± 0.01	0.71 ± 0.10	0.79 ± 0.03	0.75 ± 0.04	0.83 ± 0.03	0.84 ± 0.02	0.84 ± 0.02
	1	Absent	0.95 ± 0.01		0.81 ± 0.03	0.77 ± 0.03	0.84 ± 0.02	0.85 ± 0.03	0.85 ± 0.02
100	0	Present	0.98 ± 0.00	0.86 ± 0.06	0.89 ± 0.02	0.88 ± 0.02	0.92 ± 0.01	0.88 ± 0.02	0.89 ± 0.02
	0	Absent	0.98 ± 0.00		0.90 ± 0.01	0.88 ± 0.01	0.92 ± 0.01	0.89 ± 0.02	0.90 ± 0.02
	1	Present	0.96 ± 0.01	0.83 ± 0.07	0.85 ± 0.02	0.82 ± 0.03	0.89 ± 0.02	0.90 ± 0.02	0.89 ± 0.02
	1	Absent	0.96 ± 0.01		0.87 ± 0.02	0.83 ± 0.03	0.90 ± 0.02	0.90 ± 0.01	0.90 ± 0.01

Empirical study

- Purpose:

1. Investigate the fitness of models & analyze parameter estimates

2. Examine the validity of anchoring effects:

   analyze the relationship between item directionality / response styles

   scrutinize representative respondents

3. Further determine the robustness of anchoring effects and response styles in detecting insufficient responses:

   the effect of response times on anchoring and response styles

Empirical study

- Data set
  - the Introversion/Extraversion scale (five point scale, 1 = strongly disagree to 5 = strongly agree)
  - M = 91 items
  - 7,188 respondents (randomly choose 1,000)

  - contain items from different directions

    introversion items: ‘I am quiet around strangers’

    extraversion items: ‘I talk to a lot of different people at parties’

    single dimension: the first five eigenvalues (19.64, 3.68, 3.29, 3.18 and 2.41)

  - estimation: four chains with 10,000 (5,000 burn in)
  - convergence: $\hat{R}$ < 1.001 & the ratio N_{eff}/N > 0.5

Model fit and comparison

- Is it necessary to include various components in the model?
  - fit a series of nested models

1. Full model

$$q_{ijk} = \begin{cases} (s_j a_{jk})\theta_i + c_{jk} + \eta_{ik}, & \text{if } l_{ij} = 0 \text{ or } s_j \neq s_{l_{ij}}, \\ (s_j a_{jk})\theta_i + c_{jk} + \eta_{ik} + \rho_i \lambda_{|X_{i,\ l_{ij}} - k|}, & \text{otherwise.} \end{cases}$$

2. Content, RS & Fixed Anc

$$q_{ijk} = \begin{cases} (s_j a_{jk})\theta_i + c_{jk} + \eta_{ik}, & \text{if } l_{ij} = 0 \text{ or } s_j \neq s_{l_{ij}}, \\ (s_j a_{jk})\theta_i + c_{jk} + \eta_{ik} + \lambda_{|X_{i,\ l_{ij}} - k|}, & \text{otherwise.} \end{cases}$$

3. Content & RS

$$q_{ijk} = a_{jk}\theta_i + c_{jk} + \eta_{ik}$$

4. Content & Anc

$$q_{ijk} = \begin{cases} (s_j a_{jk})\theta_i + c_{jk}, & \text{if } l_{ij} = 0 \text{ or } s_j \neq s_{l_{ij}}, \\ (s_j a_{jk})\theta_i + c_{jk} + \rho_i \lambda_{|X_{i,\ l_{ij}} - k|}, & \text{otherwise.} \end{cases}$$

5. Content Only

$$q_{ijk} = a_{jk}\theta_i + c_{jk}$$

Model fit and comparison

- Evaluate criteria
  - relative fit:

    expected log pointwise predictive density (ELPD) based on the leave-one-out cross-validation information criterion (LOOIC)

$$\mathrm{elpd}_{\mathrm{loo}} = \sum_{i=1}^{n} \log p(\overset{\text{out-of-sample test}}{y_i} | \underset{\text{train}}{y_{-i}}) \quad \Rightarrow \quad LOOIC = -2 \times \mathrm{elpd}_{\mathrm{loo}}$$

Model fit and comparison

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➡ $LOOIC = -2 \times \mathrm{elpd}_{\mathrm{loo}}$

  - absolute fit: posterior predictive check

    empirical data (not sure whether there is an anchoring) **vs** simulated data (with anchor)

$$f_i = \frac{\sum_{j \in S_i} \left| X_{ij} - X_{i,\, l_{ij}} \right|}{\mathrm{card}\,(S_i)}$$

distance between successive item responses (calculate potential anchor)

the number of items that are in the same direction

$$S_i = \{ j \geq 2 : s_j = s_{l_{ij}} \}$$

➡ whether the simulated f_i can cover the empirical one? **The actual coverage of 95%CI & 50%CI**

Results: model fit

1. **Relative fit:**

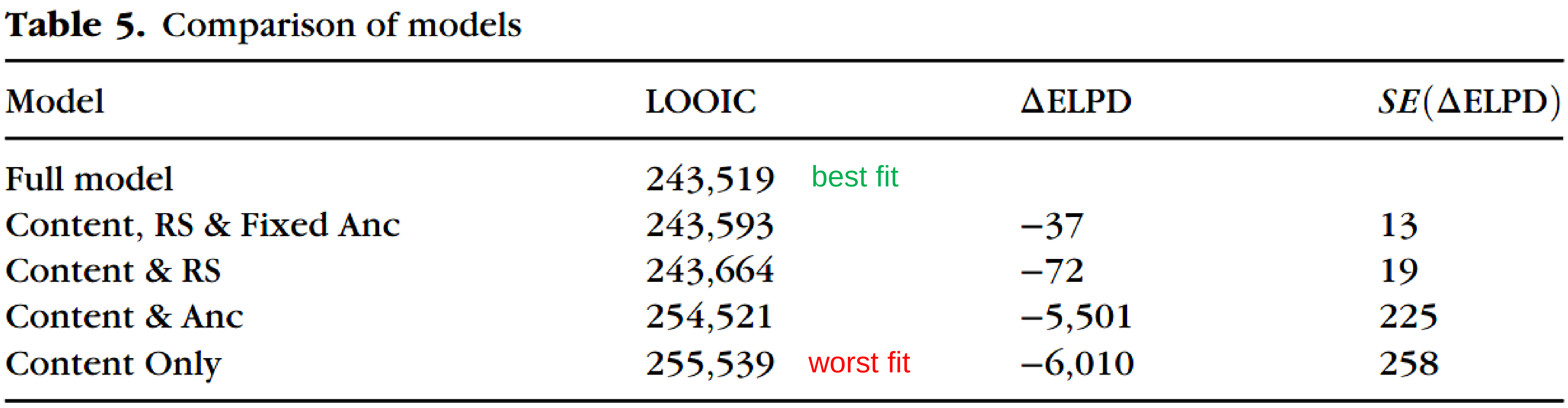

Table 5. Comparison of models

Model	LOOIC	ΔELPD	$SE(\Delta\text{ELPD})$
Full model	243,519 best fit		
Content, RS & Fixed Anc	243,593	−37	13
Content & RS	243,664	−72	19
Content & Anc	254,521	−5,501	225
Content Only	255,539 worst fit	−6,010	258

2. **Absolute fit:**

   The coverages: 55.7% (50%) and 96.0% (95%)

Results: parameter estimates (full)

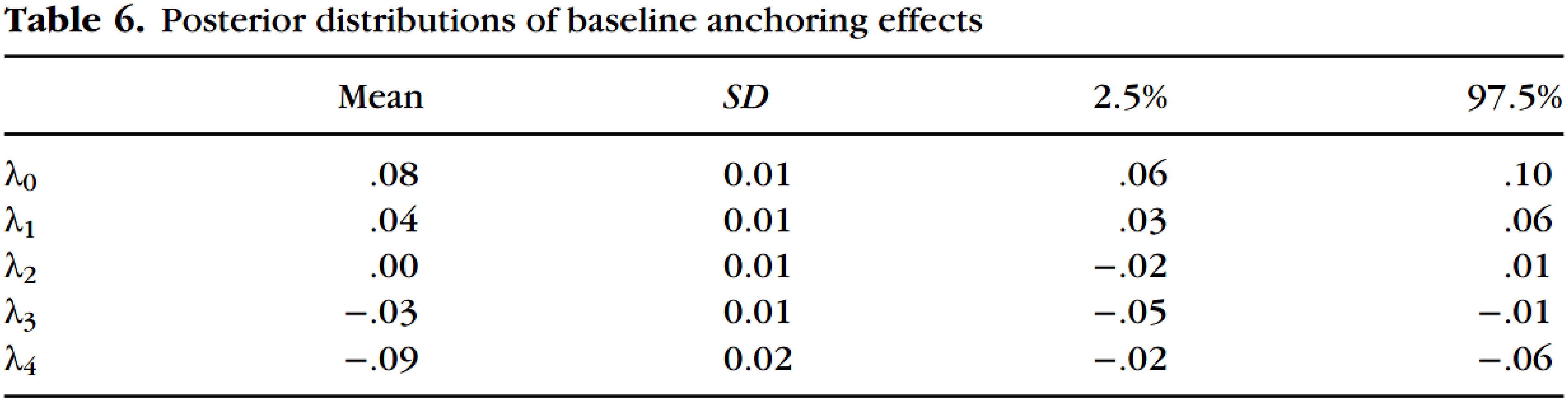

Table 6. Posterior distributions of baseline anchoring effects

	Mean	*SD*	2.5%	97.5%
λ_0	.08	0.01	.06	.10
λ_1	.04	0.01	.03	.06
λ_2	.00	0.01	−.02	.01
λ_3	−.03	0.01	−.05	−.01
λ_4	−.09	0.02	−.02	−.06

Results: parameter estimates (full)

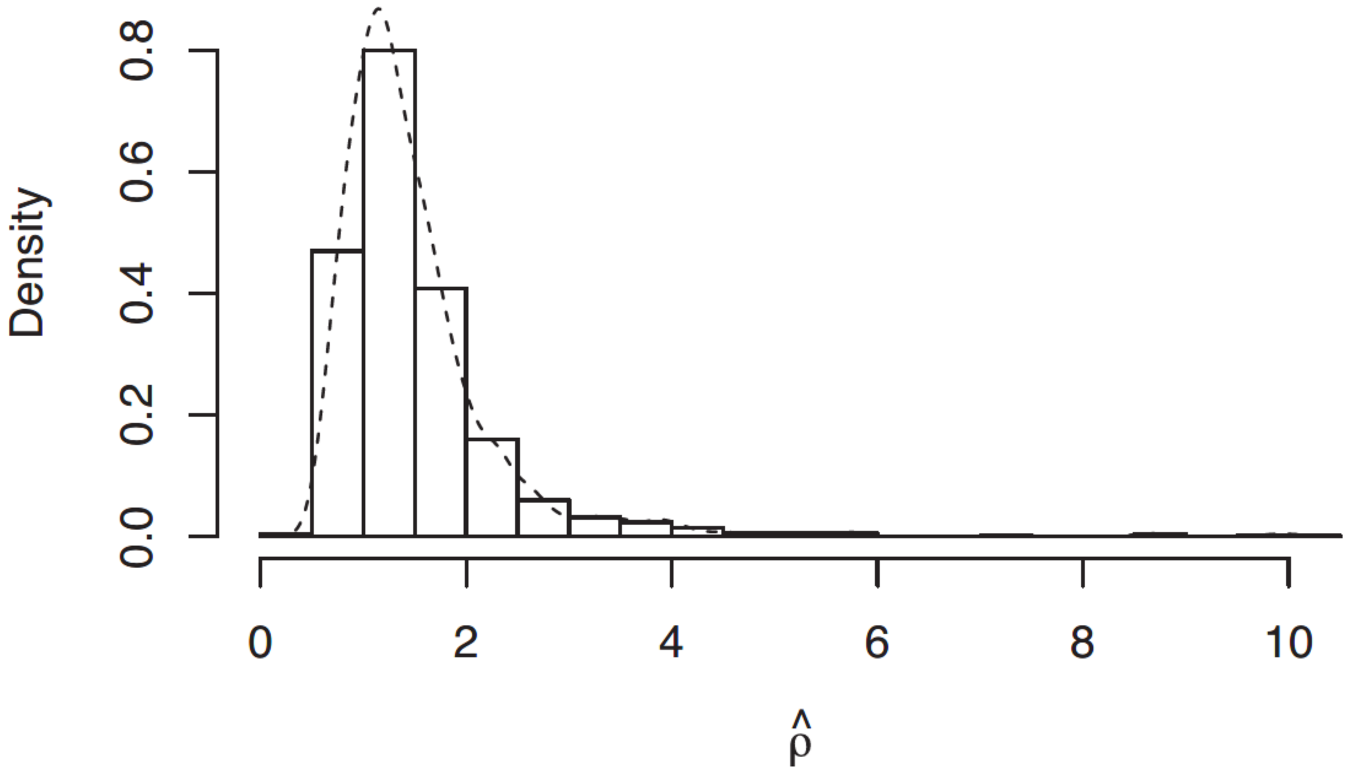

Figure 1. Histogram of strengths of anchoring effects.

Anchor & item directionality / response styles

- Does anchoring apply only to successive items of the same content / direction?
  - anchoring effect in items with opposite directions

$$q_{ijk} = \begin{cases} (s_j a_{jk})\theta_i + c_{jk} + \eta_{ik}, & \text{if } l_{ij} = 0, \\ (s_j a_{jk})\theta_i + c_{jk} + \eta_{ik} + \mu_{|X_{i,l_{ij}}-k|}, & \text{if } l_{ij} \geq 1 \text{ and } s_j \neq s_{l_{ij}}, \quad \text{opposite direction} \\ (s_j a_{jk})\theta_i + c_{jk} + \eta_{ik} + \lambda_{|X_{i,l_{ij}}-k|}, & \text{otherwise}, \quad \text{same direction} \end{cases}$$

(the respondent-specific anchoring parameter ρ_i is omitted)

- What is the connection between anchoring effect and response styles?

Results: item directionality

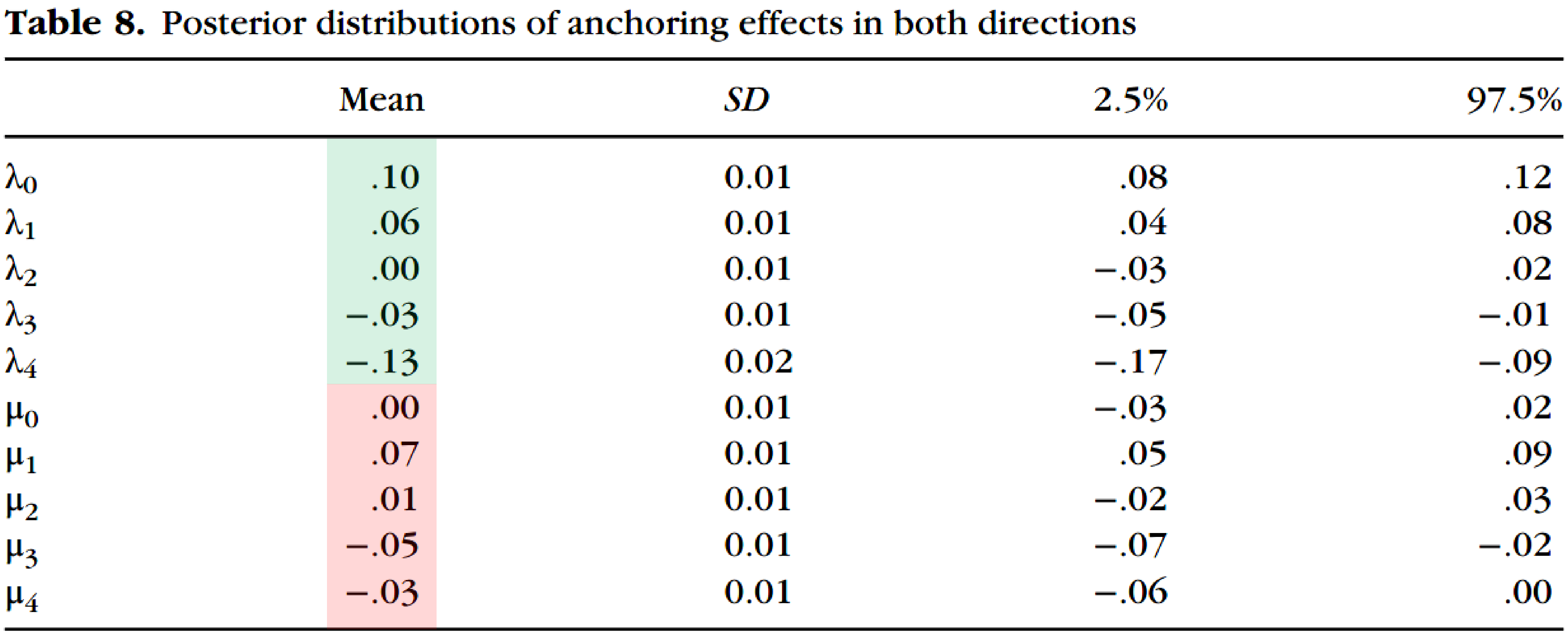

Table 8. Posterior distributions of anchoring effects in both directions

	Mean	*SD*	2.5%	97.5%
λ_0	.10	0.01	.08	.12
λ_1	.06	0.01	.04	.08
λ_2	.00	0.01	−.03	.02
λ_3	−.03	0.01	−.05	−.01
λ_4	−.13	0.02	−.17	−.09
μ_0	.00	0.01	−.03	.02
μ_1	.07	0.01	.05	.09
μ_2	.01	0.01	−.02	.03
μ_3	−.05	0.01	−.07	−.02
μ_4	−.03	0.01	−.06	.00

Results: response styles

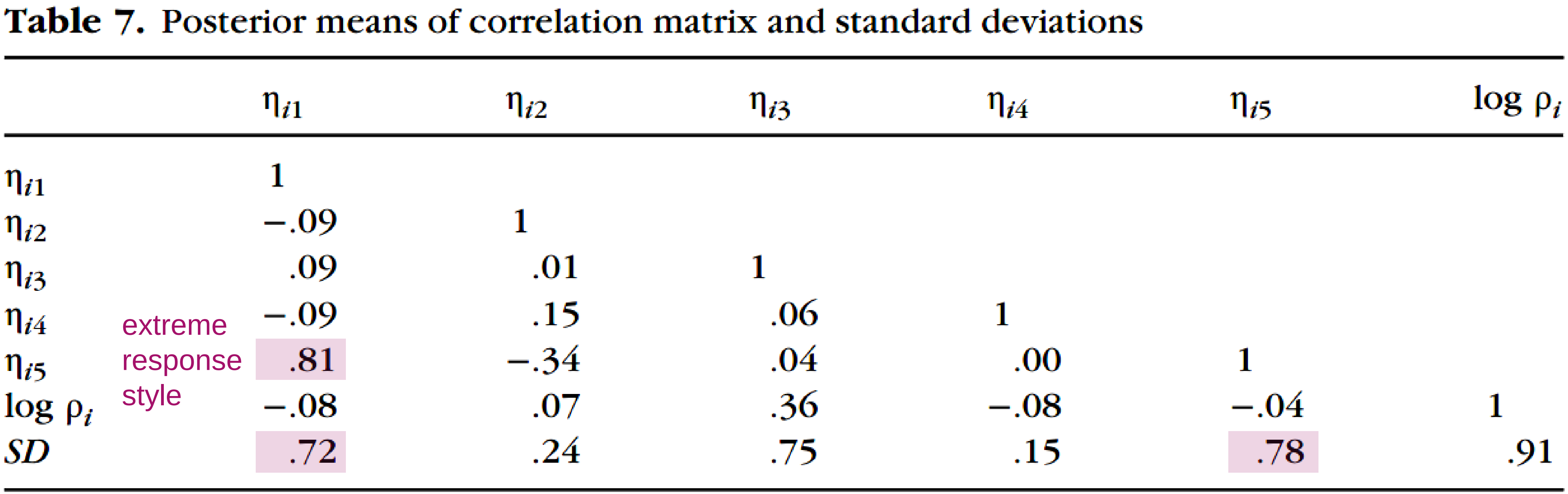

Table 7. Posterior means of correlation matrix and standard deviations

	η_{i1}	η_{i2}	η_{i3}	η_{i4}	η_{i5}	$\log \rho_i$
η_{i1}	1					
η_{i2}	−.09	1				
η_{i3}	.09	.01	1			
η_{i4}	−.09	.15	.06	1		
η_{i5}	.81	−.34	.04	.00	1	
$\log \rho_i$	−.08	.07	.36	−.08	−.04	1
SD	.72	.24	.75	.15	.78	.91

extreme response style

avoid extreme

avoid extreme

Results: response styles

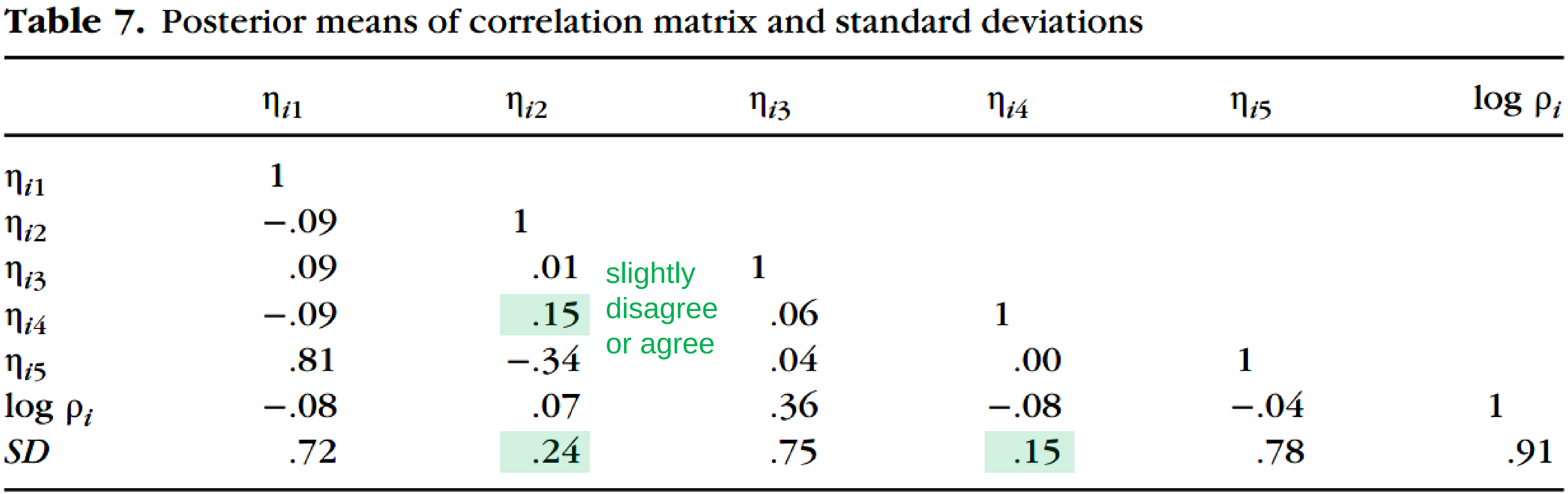

Table 7. Posterior means of correlation matrix and standard deviations

	η_{i1}	η_{i2}	η_{i3}	η_{i4}	η_{i5}	$\log \rho_i$
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η_{i2}	−.09	1				
η_{i3}	.09	.01	1			
η_{i4}	−.09	.15	.06	1		
η_{i5}	.81	−.34	.04	.00	1	
$\log \rho_i$	−.08	.07	.36	−.08	−.04	1
SD	.72	.24	.75	.15	.78	.91

slightly disagree or agree

Results: response styles

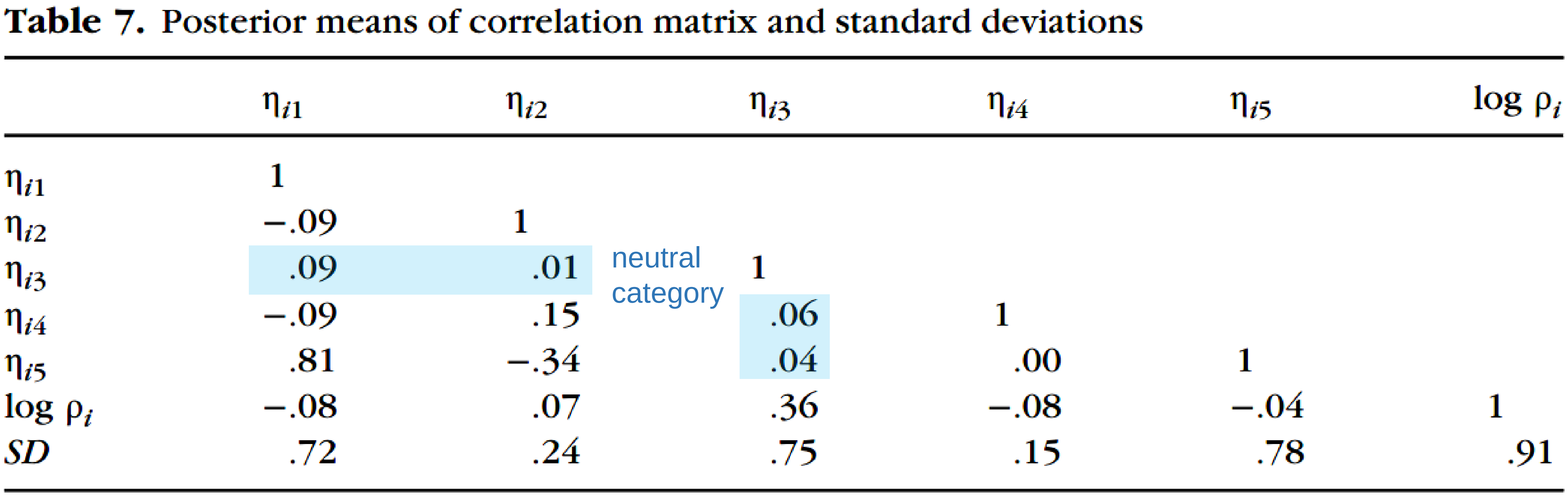


Table 7. Posterior means of correlation matrix and standard deviations

	η_{i1}	η_{i2}	η_{i3}	η_{i4}	η_{i5}	$\log \rho_i$
η_{i1}	1					
η_{i2}	−.09	1				
η_{i3}	.09	.01	1			
η_{i4}	−.09	.15	.06	1		
η_{i5}	.81	−.34	.04	.00	1	
$\log \rho_i$	−.08	.07	.36	−.08	−.04	1
SD	.72	.24	.75	.15	.78	.91

neutral category

Results: response styles

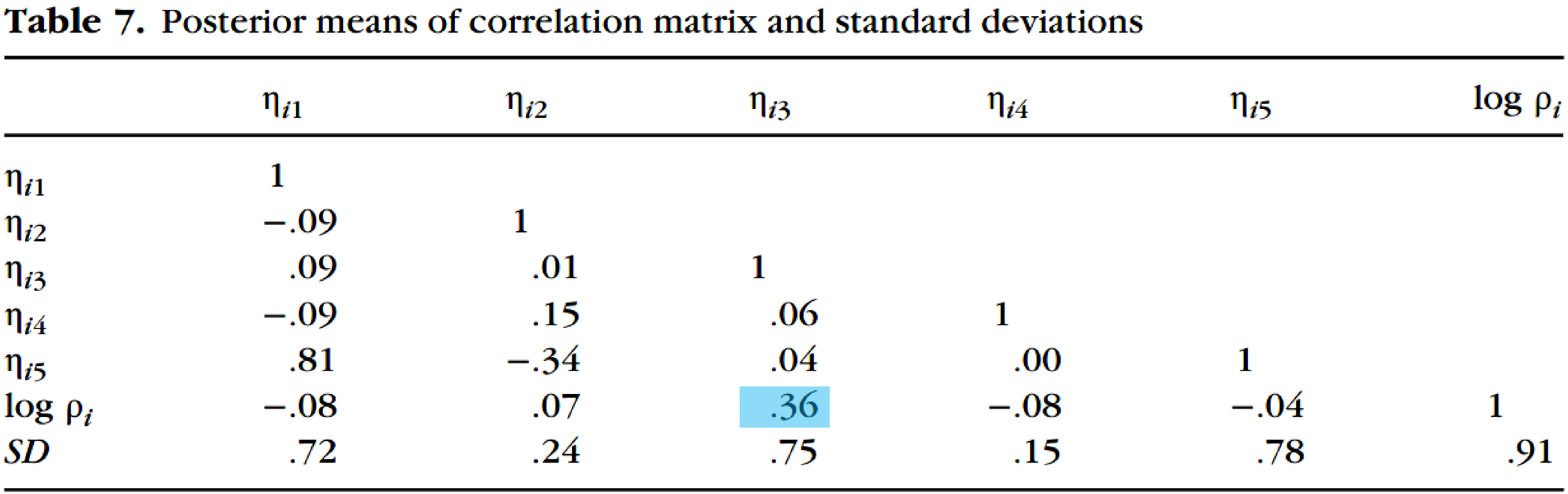
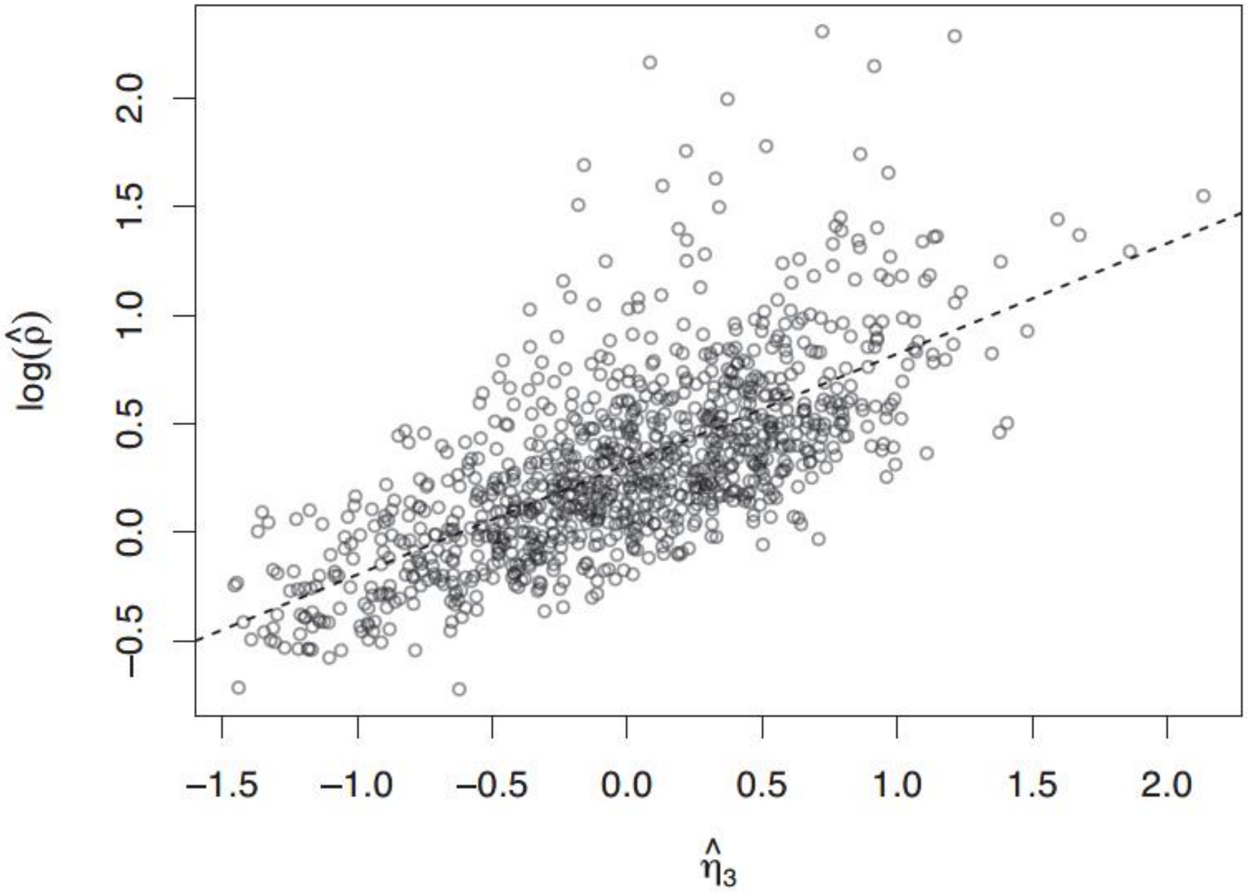


Table 7. Posterior means of correlation matrix and standard deviations

	η_{i1}	η_{i2}	η_{i3}	η_{i4}	η_{i5}	$\log \rho_i$
η_{i1}	1					
η_{i2}	−.09	1				
η_{i3}	.09	.01	1			
η_{i4}	−.09	.15	.06	1		
η_{i5}	.81	−.34	.04	.00	1	
$\log \rho_i$	−.08	.07	.36	−.08	−.04	1
SD	.72	.24	.75	.15	.78	.91

Both anchoring and response style can be manifestations of reduced respondent effort?

Results: response styles



(reduced efforts)

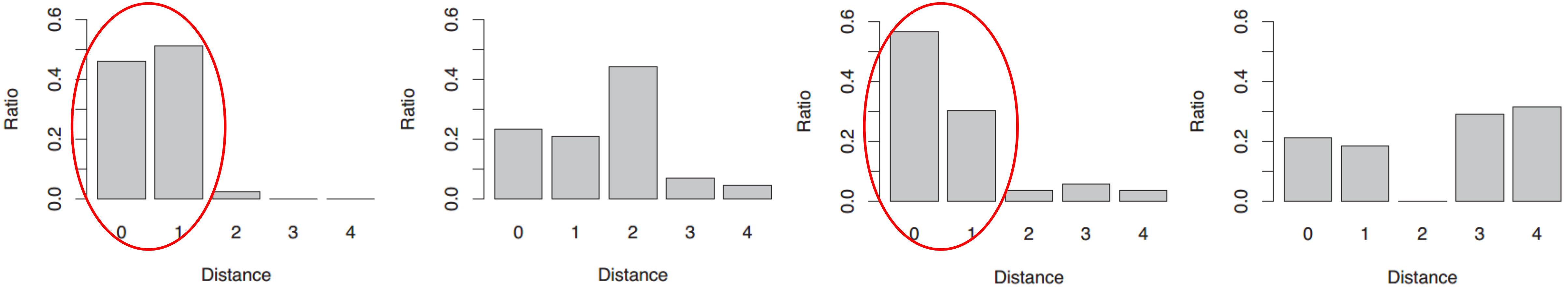
(an unwillingness to make a decision)

Figure 2. Scatter plot of strengths of anchoring effects against midpoint response styles.

Results: response styles

Table 9. Response patterns of representative respondents

No.	$\hat{\eta}_3$	$\hat{\rho}$	$\hat{\theta}$	Successive introversion items	Successive extraversion items
12	2.13	4.71	0.07	4,4,3,3,3,2,2,1	3,2,3,3,3
523	1.11	1.44	0.68	4,3,5,3,5	3,1,3,1,1
15	−0.47	2.04	1.37	4,4,5,4,5,5,4,5,5,5,2	1,4,4,4,3,2,1,1,1
265	−1.44	0.49	−0.46	1,4,1,2,5,1	1,4,1,5,5



(a) 12 (High MRS, Strong Anchoring) **(b)** 523 (High MRS, Weak Anchoring) **(c)** 15 (Low MRS, Strong Anchoring) **(d)** 265 (Low MRS, Weak Anchoring)

Figure 3. Absolute differences of consecutive responses for representative respondents. (MRS = midpoint response styles)

Relationship with response time

- Can response style and anchoring really reflect reduced respondent motivation?
  - Hypothesis:

    reduced effort = shorter response time +
    stronger response styles +
    stronger anchoring behaviour

  - Model with response time:

$$t_{ij} = \frac{\mathrm{time}_{ij}}{\frac{1}{M}\sum_{m=1}^{M}\mathrm{time}_{im}}$$

(standardization: different respondents spend different times)

$$q_{ijk} = \begin{cases} (s_j a_{jk})\theta_i + c_{jk} + t_{ij}^{\alpha_{\mathrm{RS}}}\eta_{ik}, & \text{if } l_{ij} = 0 \text{ or } s_j \neq s_{l_{ij}}, \\ (s_j a_{jk})\theta_i + c_{jk} + t_{ij}^{\alpha_{\mathrm{RS}}}\eta_{ik} + \rho_i t_{ij}^{\alpha_{\mathrm{AE}}}\lambda^{\left|X_{i,l_{ij}} - k\right|}, & \text{otherwise,} \end{cases}$$

measure the association with response time
(strengthen or weaken the response styles / anchoring)

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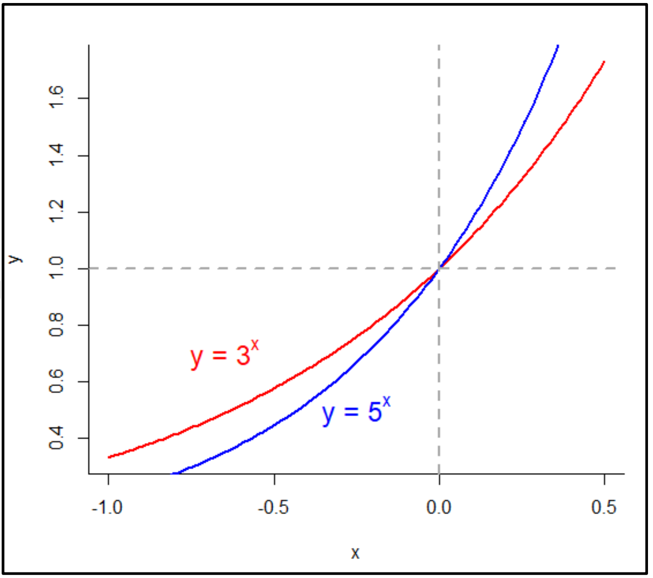
(standardization: different respondents spen

$$q_{ijk} = \begin{cases} (s_j a_{jk})\theta_i + c_{jk} + t_{ij}^{\alpha_{\mathrm{RS}}}\eta_{ik}, & \text{if } l_{ij} = 0 \text{ or } s_j \neq s_{l_{ij}}, \\ (s_j a_{jk})\theta_i + c_{jk} + t_{ij}^{\alpha_{\mathrm{RS}}}\eta_{ik} + \rho_i t_{ij}^{\alpha_{\mathrm{AE}}}\lambda_{|X_{i,l_{ij}} - k|}, & \text{otherwise,} \end{cases}$$

measure the association with response time
(strengthen or weaken the response styles / anchoring)

  - Estimation: priors $\mathcal{N}(0, 2)$

the shorter, the larger tendency



➡ both should be negative

Relationship with response time

- Response time data and preprocessing
  - response times:

    1% and 99% quantiles = 1.067 and 26.018 second

  - winsorization:

    763 response times that are >30 s were set at exactly 30 s

    (e.g., respondents who left the computer for a period of time before coming back)

Results

‘I would have to be lost for a very long time before asking help’

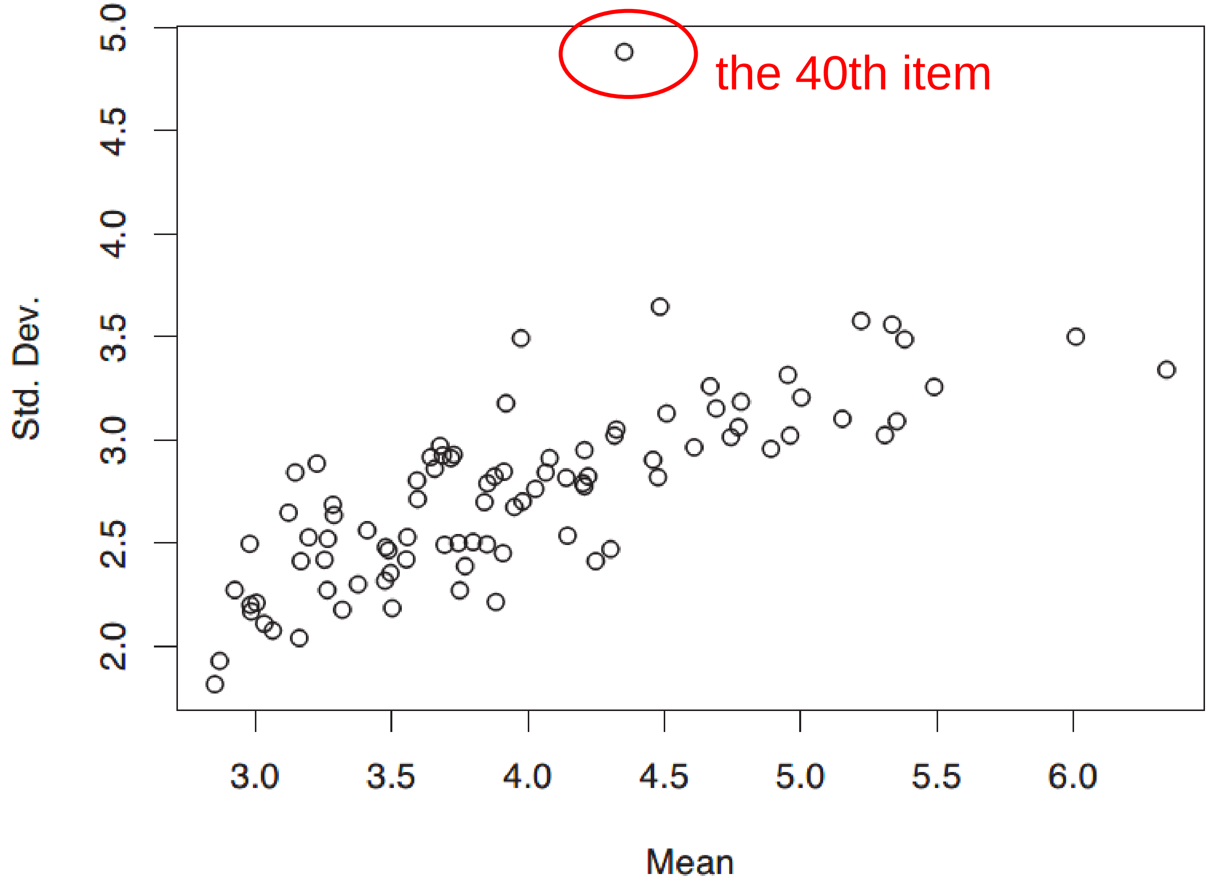


Figure 4. Scatter plot of mean and standard deviation of response times for each item.

Results

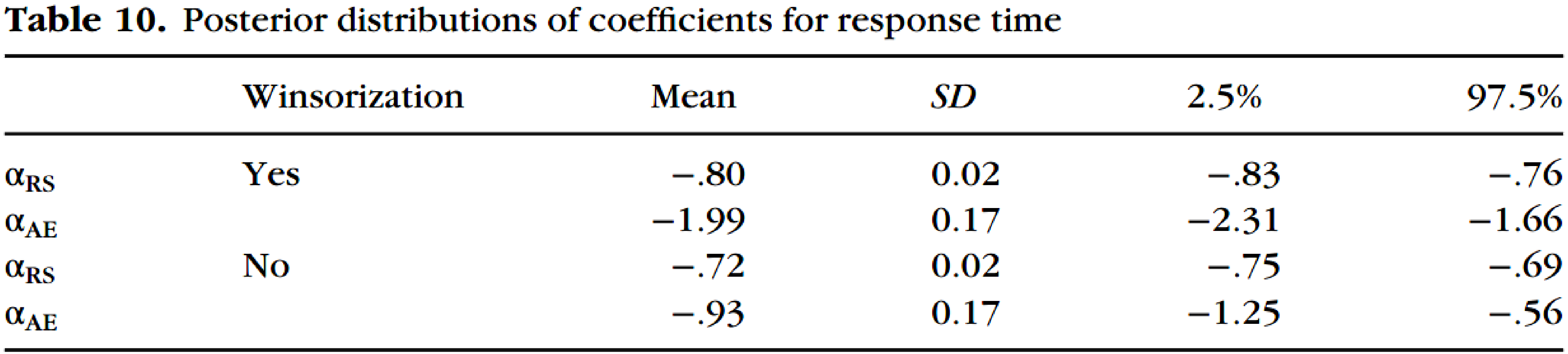


Table 10. Posterior distributions of coefficients for response time

	Winsorization	Mean	*SD*	2.5%	97.5%
α_{RS}	Yes	−.80	0.02	−.83	−.76
α_{AE}		−1.99	0.17	−2.31	−1.66
α_{RS}	No	−.72	0.02	−.75	−.69
α_{AE}		−.93	0.17	−1.25	−.56

Discussion and limitations

- facilitate the design of rating scale instruments:
  - remove a midpoint category
  - intentionally intermixing items of different content types or widely varying mean scores

- the test-taking population: volunteers (more engaged)
- if the items possess a high level of parallelism, anchoring and response styles could become more easily confounded
- strengthening of both anchoring effects and response styles for items later in the test
- incorporate the content trait into the multivariate distribution assumed for respondent anchoring and response style parameters

The End. Thanks for Listening!

谢谢大家

多谢晒~

ありがとう

Danke

Merci

Reporter: Yingshi Huang