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On Longitudinal Item Response Theory Models: A Didactic

Chun Wang
University of Washington

Steven W. Nydick
Duolingo

Reporter: Yingshi Huang

Introduction

- How to determine student growth?

This student is learning math

He took four exams in his grade 3, 4, 5, and 6

Responses 1	Responses 2	Responses 3	Responses 4
↓	↓	↓	↓
ability at time 1	ability at time 2	ability at time 3	ability at time 4

- We want to model the changes in latent traits

  from item response theory (IRT) model → single time point

  to longitudinal IRT models (L-IRT) → multiple time points

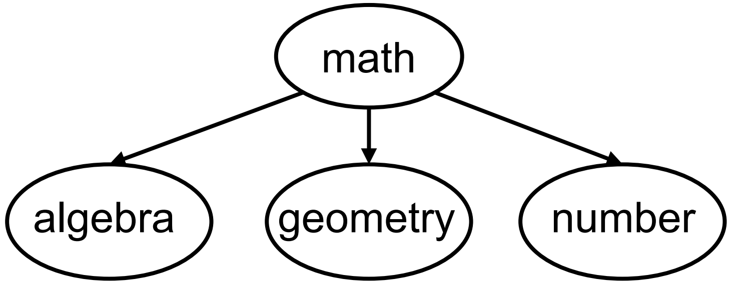
Introduction

- The family of longitudinal IRT
  - measurement model
  - the relationship of the latent traits over time

Introduction

- The family of longitudinal IRT
  - measurement model
  - the relationship of the latent traits over time

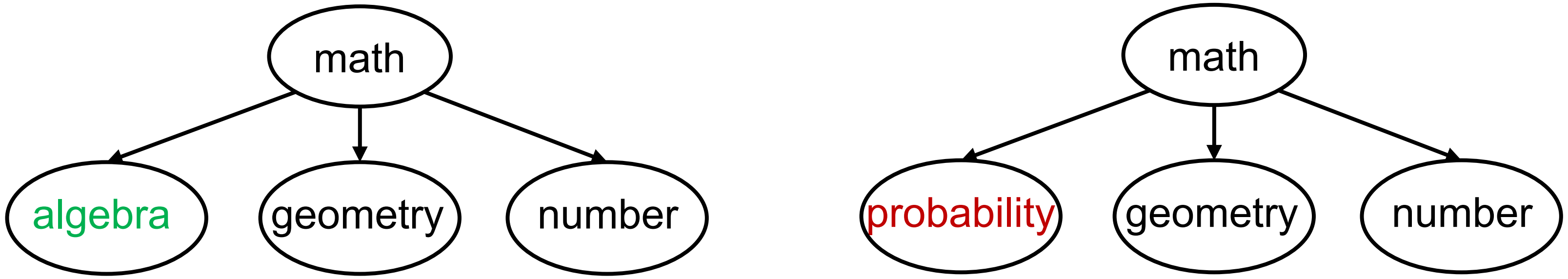
1. Unidimensional model: one latent trait (e.g., math ability)
2. Multidimensional model: multiple latent traits (e.g., math, reading, science)
3. Hierarchical model



Introduction

- The family of longitudinal IRT
  - measurement model
  - the relationship of the latent traits over time

3. Hierarchical model



- Content coverage shifts across times:
  many domains only appear in limited grades
- Different growth patterns:
  certain domain-level traits grow linearly, whereas others grow in a piecewise fashion

Introduction

- The family of longitudinal IRT
  - measurement model
  - the relationship of the latent traits over time

1. unstructured covariance matrix

ability_t1 ability_t2 ability_t3 ability_t4

ability_t1

ability_t2

ability_t3

ability_t4

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1T} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} & \cdots & \sigma_{2T} \\ \sigma_{13} & \sigma_{12} & \sigma_3^2 & \cdots & \sigma_{3T} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{1T} & \sigma_{2T} & \sigma_{3T} & \cdots & \sigma_T^2 \end{pmatrix}$$

2. latent growth curve (LGC) models

   ability at time t
   = intercept + slope*time effect + error

Introduction

- The family of longitudinal IRT
  - measurement model
  - the relationship of the latent traits over time

- This paper:

  introduce **three models** with the **latent growth curve structure**

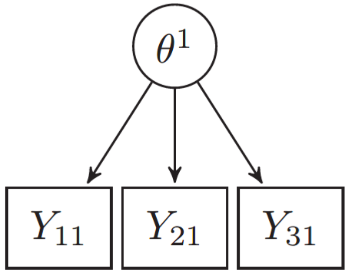
  (LGC is more interpretable & more complicated than unstructured covariance matrix)

1. Longitudinal unidimensional IRT
2. Longitudinal multidimensional IRT
3. Longitudinal higher-order IRT

Longitudinal IRT models

- Longitudinal unidimensional IRT (L-UIRT) model
  - One time point:

$$\Pr\left(Y_{ij} = 1 \middle| \theta_i, a_j, b_j\right) = \frac{1}{1 + \exp\left[-a_j\left(\theta_i - b_j\right)\right]}$$



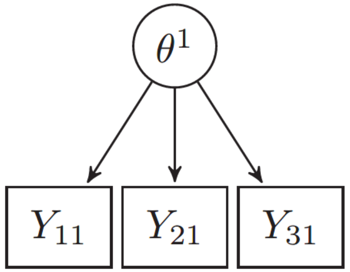
Math ability

Items

Longitudinal IRT models

- Longitudinal unidimensional IRT (L-UIRT) model
  - One time point:

$$\Pr\left(Y_{ij} = 1 \middle| \theta_i, a_j, b_j\right) = \frac{1}{1 + \exp\left[-a_j\left(\theta_i - b_j\right)\right]}$$

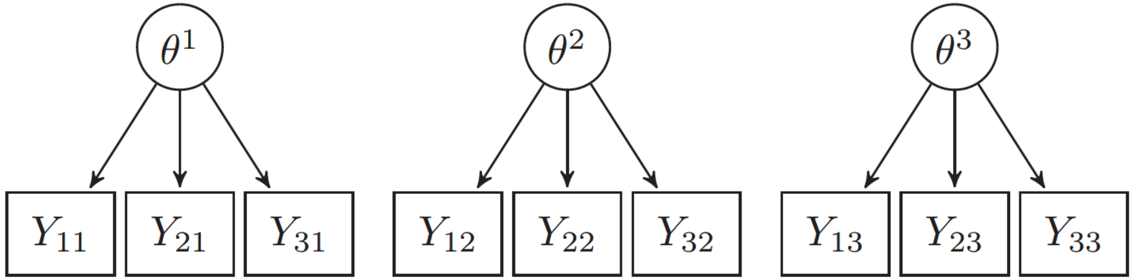


Math ability

Items

  - Multiple time points:

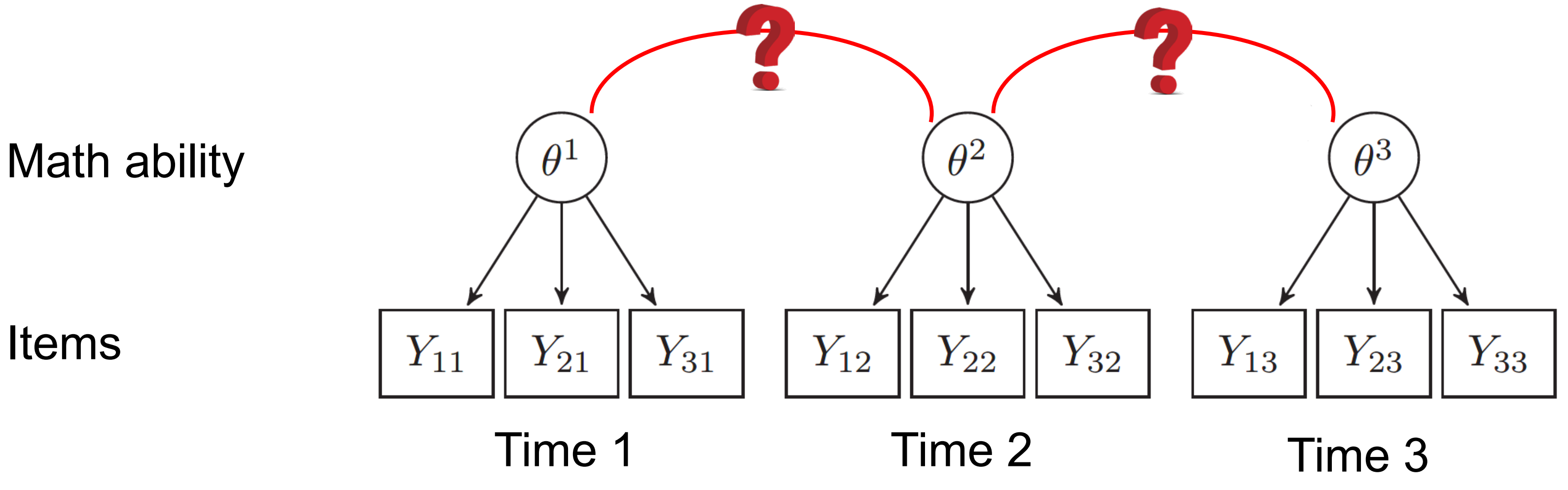
$$\Pr\left(Y_{ij}^t = 1 \middle| \theta_i^t, a_j^t, b_j^t\right) = \frac{1}{1 + \exp\left[-a_j^t\left(\theta_i^t - b_j^t\right)\right]}$$



Time 1 Time 2 Time 3

Longitudinal IRT models

- The relationship of the latent traits over time



Design matrix (T-by-p and T-by-q)

$$\boldsymbol{\theta}_i = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\nu}_i + \boldsymbol{\delta}_i$$

Fixed effects (p)

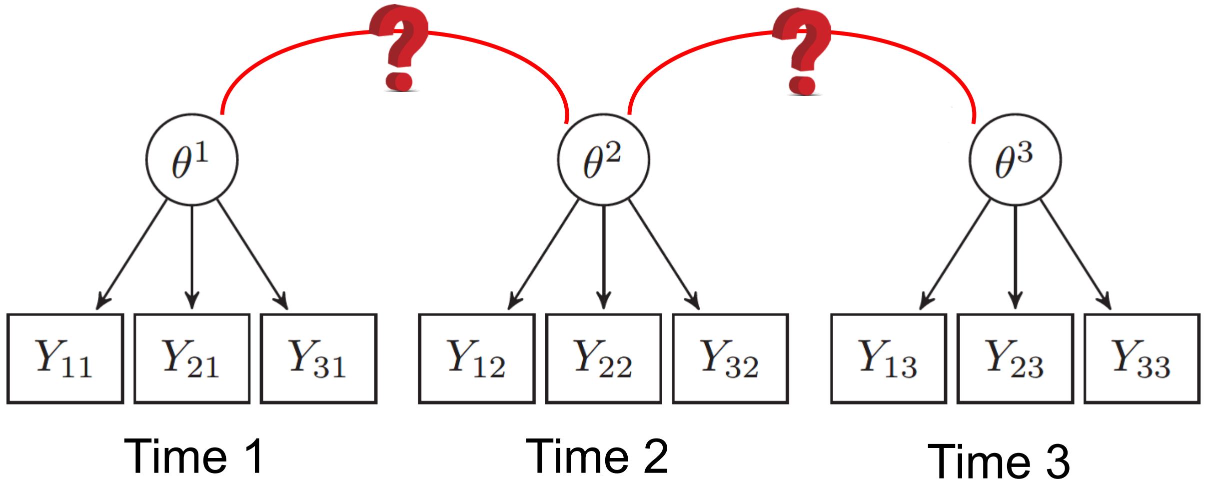
Random effects (q)

$\boldsymbol{\theta}_i = \{\theta_i^1, \dots, \theta_i^t, \dots, \theta_i^T\}$ (T-by-1 vector)

$\boldsymbol{\delta}_i$ Residuals (T-by-1 vector)

Longitudinal IRT models

- The relationship of the latent traits over time



Design matrix (T-by-p and T-by-q)

$$\boldsymbol{\theta}_i = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{v}_i + \boldsymbol{\delta}_i$$

Fixed effects (p)

Random effects (q)

$\boldsymbol{\theta}_i = \{\theta_i^1, \ldots, \theta_i^t, \ldots, \theta_i^T\}$ (T-by-1 vector)

$\boldsymbol{\delta}_i$ Residuals (T-by-1 vector)

Example:
a simple linear growth model with a single person-specific intercept and slope ($p = q = 2$)

$$\mathbf{X} = \mathbf{Z} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & T-1 \end{pmatrix}$$

$$\theta_i^t = \pi_{0i} + \pi_{1i} \times (t-1) + \delta_i^t$$

$$\pi_{0i} = \beta_0 + \nu_{0i}$$

$$\pi_{1i} = \beta_1 + \nu_{1i}$$

$$\boldsymbol{\beta} = \{\beta_0, \beta_1\} \qquad \boldsymbol{v} = \{v_{0i}, v_{1i}\} \sim \text{multinormal}(\mathbf{0}, \Sigma_v)$$

$$\Sigma_v = \begin{bmatrix} \sigma_{v_0}^2 & \sigma_{v_0 v_1} \\ \sigma_{v_0 v_1} & \sigma_{v_1}^2 \end{bmatrix}$$

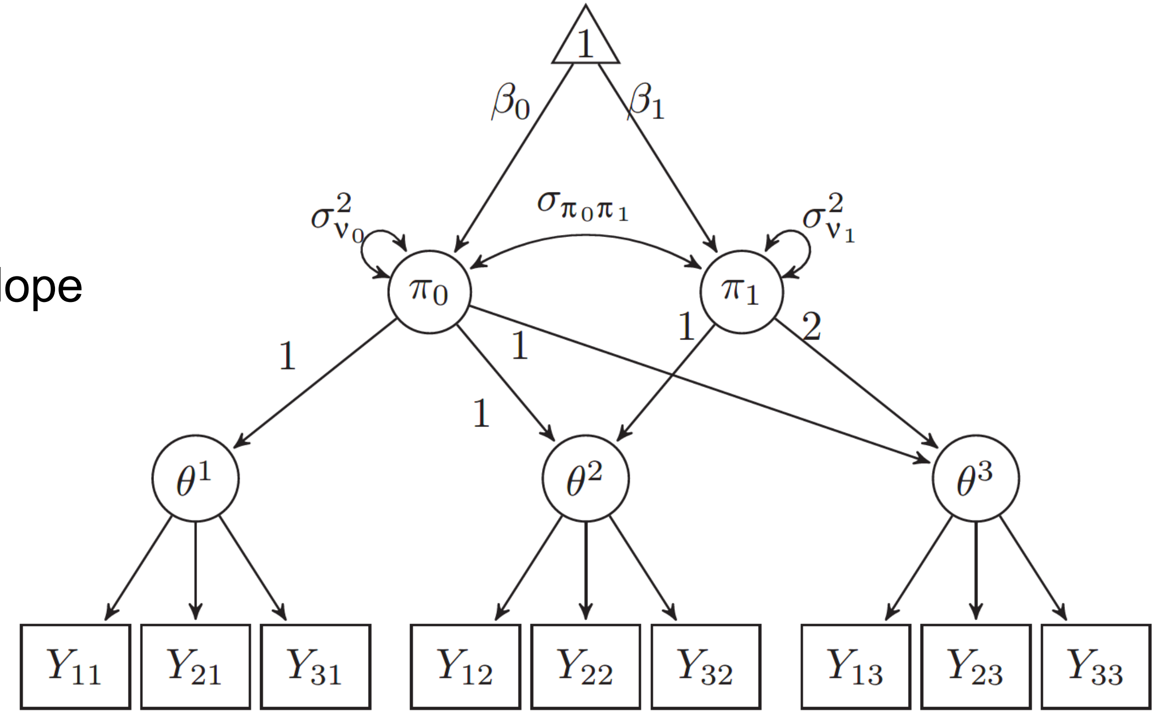
Longitudinal IRT models

- Three time points, one latent trait

Intercept and slope

Math ability

Items



$$\theta_i^t = \pi_{0i} + \pi_{1i} \times (t - 1) + \delta_i^t$$

$$\pi_{0i} = \beta_0 + \nu_{0i}$$

$$\pi_{1i} = \beta_1 + \nu_{1i}$$

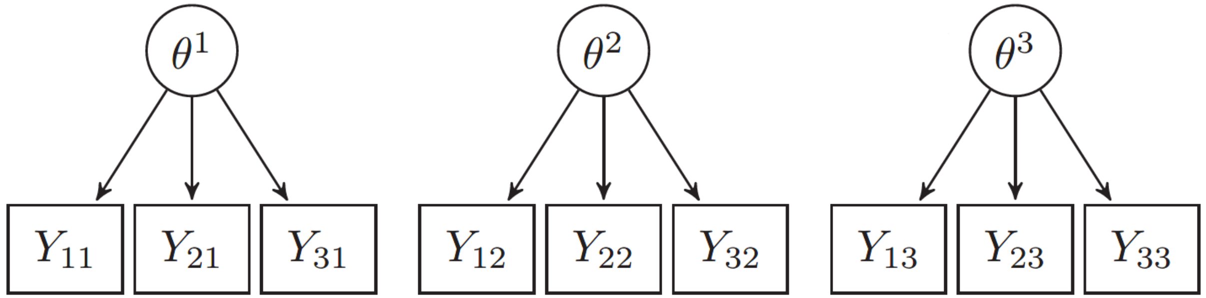
$$\Pr\left(Y_{ij}^t = 1 | \theta_i^t, a_j^t, b_j^t\right) = \frac{1}{1 + \exp\left[-a_j^t\left(\theta_i^t - b_j^t\right)\right]}$$

Longitudinal IRT models

- The scale of parameters: linking

Math ability

Items



Time 1 Time 2 Time 3

– Different items may be used at different time points

$$\Pr\left(Y_{ij}^t = 1 \mid \theta_i^t, a_j^t, b_j^t\right)$$

Person ability Item parameters

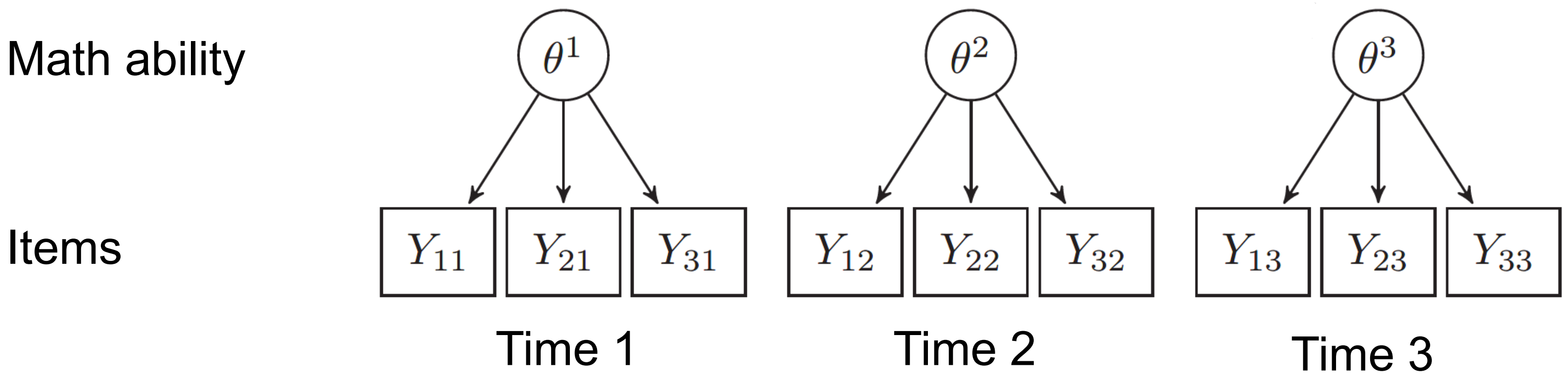
Low ability + easy question

High ability + hard question

➡ Same result

Longitudinal IRT models

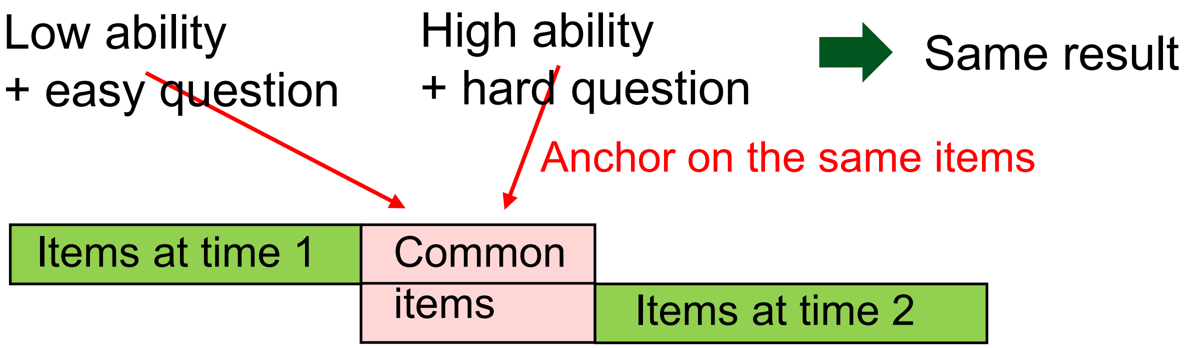
- The scale of parameters: linking



– Different items may be used at different time points

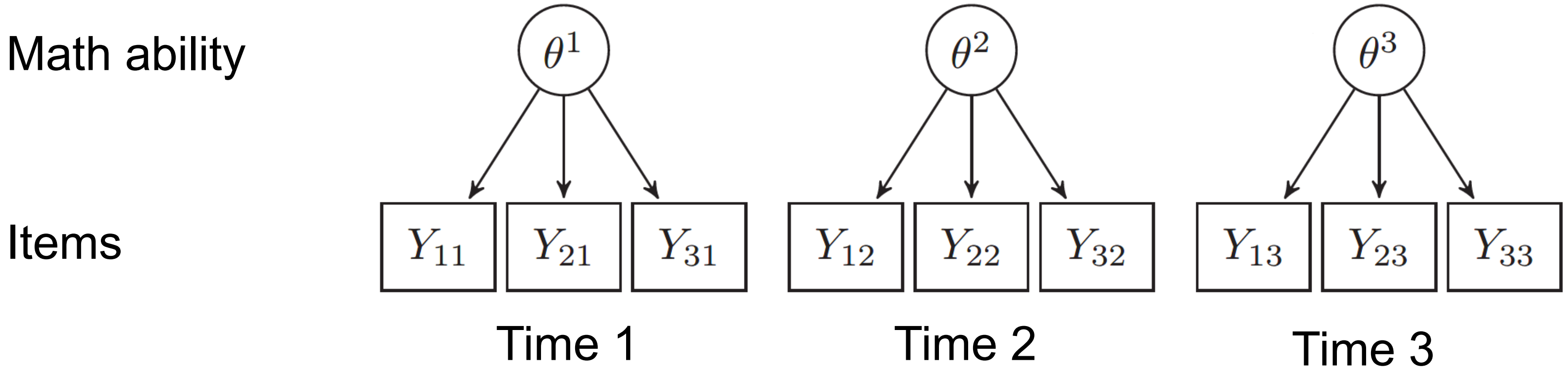
$$\Pr\left(Y_{ij}^t = 1 | \theta_i^t, a_j^t, b_j^t\right)$$

Person ability   Item parameters



Longitudinal IRT models

- The scale of parameters: identifiability



– Within a specific time point: item part and person part still mix together

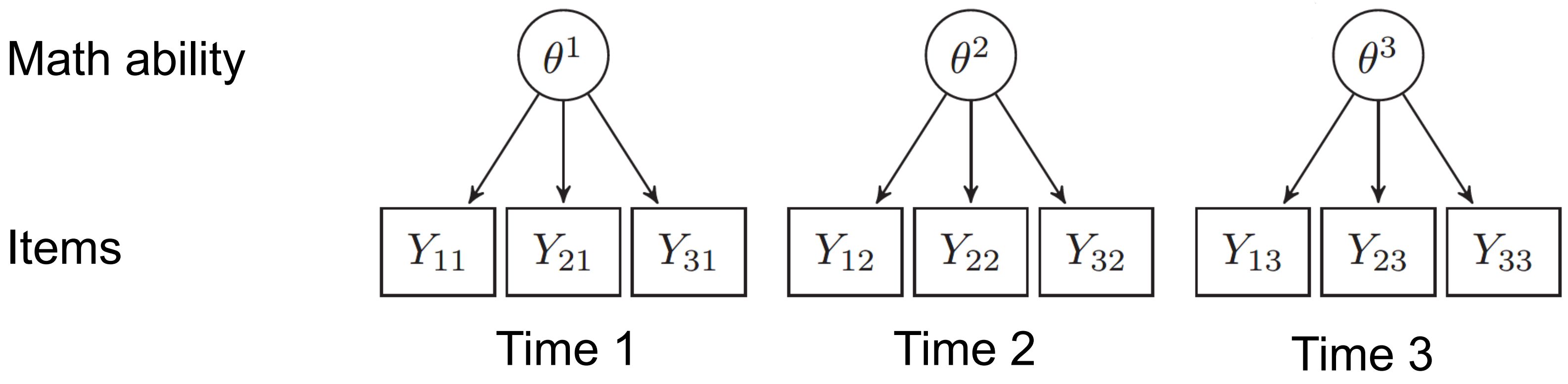
$$\Pr\left(Y_{ij}^t = 1 | \theta_i^t, a_j^t, b_j^t\right) = \frac{1}{1 + \exp\left[-a_j^t\left(\theta_i^t - b_j^t\right)\right]}$$

➡ Impose some constraints:
fix the scale of one part at one time point (person ability at time 1)

1. mean [= 0]
2. variance [= constant]

Longitudinal IRT models

- The scale of parameters: identifiability



– Within a specific time point: item part and person part still mix together

$$\Pr\left(Y_{ij}^t = 1 | \theta_i^t, a_j^t, b_j^t\right) = \frac{1}{1 + \exp\left[-a_j^t\left(\theta_i^t - b_j^t\right)\right]}$$

➡ Impose some constraints:
fix the scale of one part at one time point (person ability at time 1)

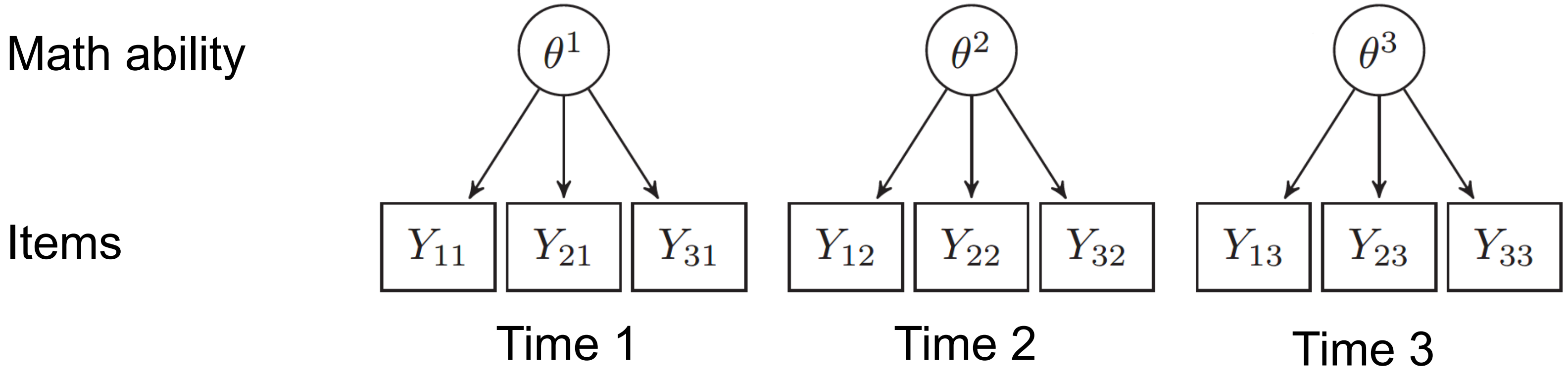
1. mean [= 0]
2. variance [= constant]

All of the residuals having mean 0 $E(\delta_i^t) = 0$

$$\theta_i^t = \pi_{0i} + \pi_{1i} \times (t-1) + \delta_i^t$$
$$\pi_{0i} = \beta_0 + \nu_{0i}$$
$$\pi_{1i} = \beta_1 + \nu_{1i}$$

Longitudinal IRT models

- The scale of parameters: identifiability



– Within a specific time point: item part and person part still mix together

$$\Pr\left(Y_{ij}^t = 1 | \theta_i^t, a_j^t, b_j^t\right) = \frac{1}{1 + \exp\left[-a_j^t\left(\theta_i^t - b_j^t\right)\right]}$$

➡ Impose some constraints:
fix the scale of one part at one time point (person ability at time 1)

1. mean [= 0]
2. variance [= constant]

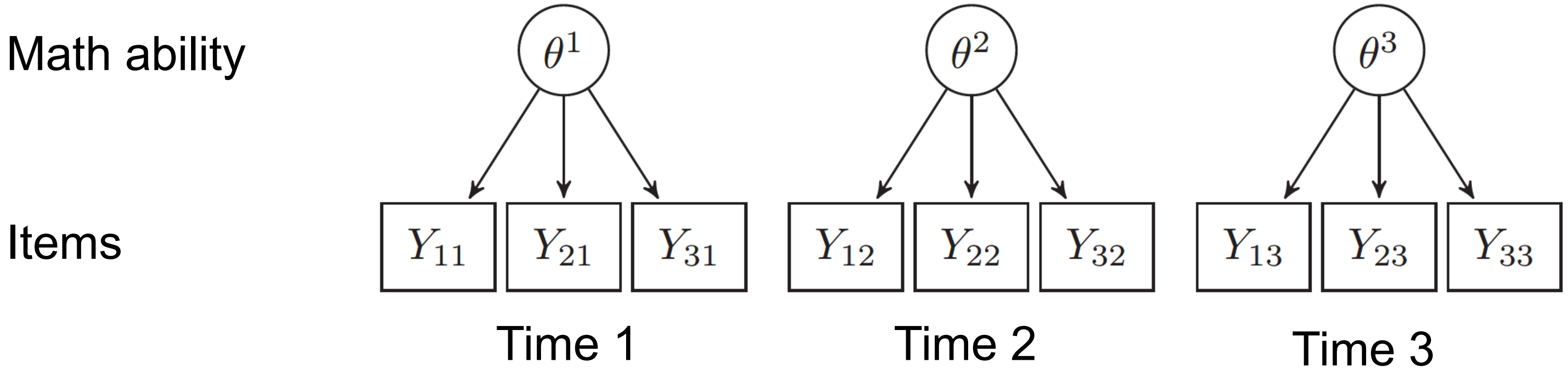
mean — variance

$$\theta_i^t = \pi_{0i} + \pi_{1i} \times (t-1) + \delta_i^t$$

$$\mu_{\pi_{0i}} = \beta_0 = 0 \qquad \sigma^2_{\delta_i^{(1)}} = c_1$$

Longitudinal IRT models

- The scale of parameters: identifiability



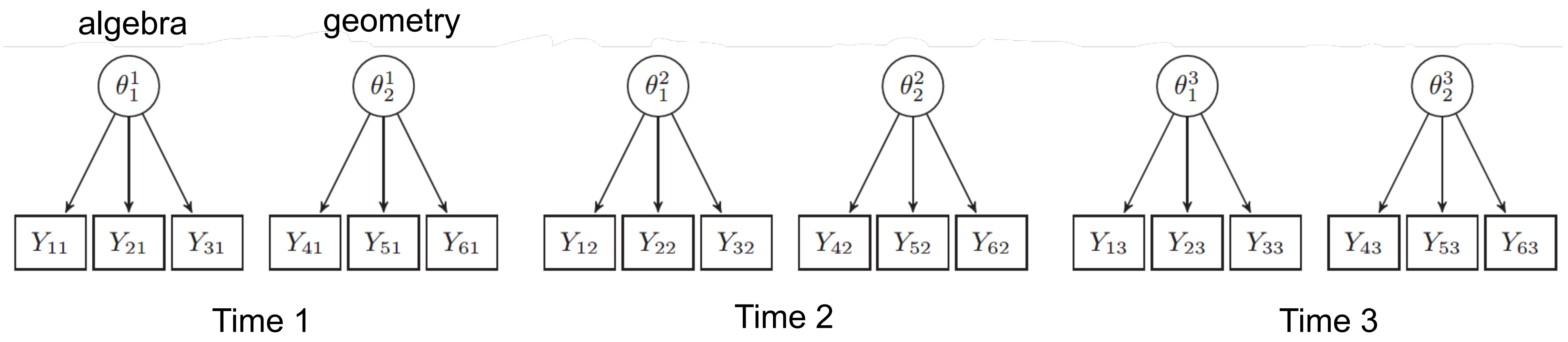
– Within a specific time point: item part and person part still mix together

$$\Pr\left(Y_{ij}^t = 1 | \theta_i^t, a_j^t, b_j^t\right) = \frac{1}{1 + \exp\left[-a_j^t\left(\theta_i^t - b_j^t\right)\right]}$$

➡ If the anchor items have been pre-calibrated (i.e., we have a scale of item parameters)
Then only the constraint of expected δ_i^t is needed $E(\delta_i^t) = 0$

Longitudinal IRT models

- Longitudinal multidimensional IRT (L-MIRT) model



$$\Pr\left(Y_{ij}^t = 1 | \theta_i^t, a_j^t, b_j^t\right) = \frac{1}{1 + \exp\left[-a_j^t\left(\theta_i^t - b_j^t\right)\right]}$$

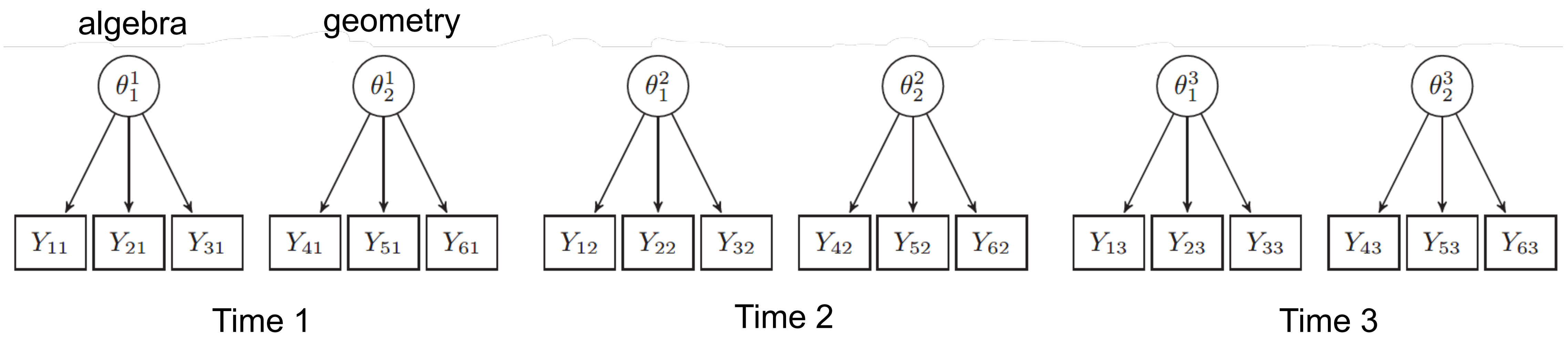
→

$$\Pr\left(Y_{ij}^t = 1 | \boldsymbol{\theta}_i^t, a_j^t, b_j^t\right) = \frac{1}{1 + \exp\left[-\left(\mathbf{a}_j^t\right)^{\mathrm{T}} \boldsymbol{\theta}_i^t + b_j^t\right]}$$

More than one ability were involved (K dimensions)

Longitudinal IRT models

- The relationship of the latent traits over time

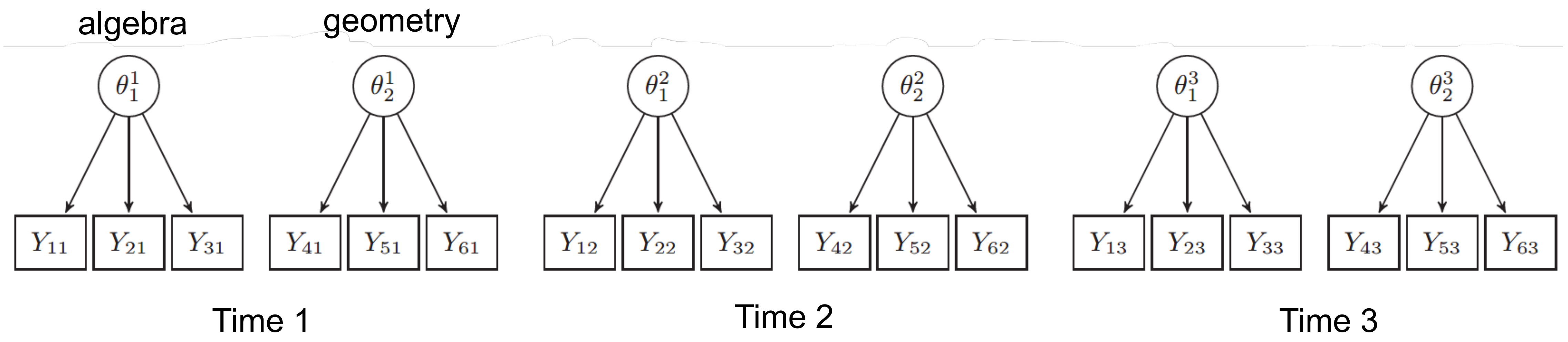


$$\boldsymbol{\theta}_i = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\nu}_i + \boldsymbol{\delta}_i$$

$$\boldsymbol{\theta}_i = (\underbrace{\theta_{i1}^1, \ldots, \theta_{iK}^1}_{K \text{ dimensions}}, \ldots, \theta_{i1}^T, \ldots, \theta_{iK}^T)' \quad KT\text{-by-1 vector}$$

Longitudinal IRT models

- The relationship of the latent traits over time



$$\boldsymbol{\theta}_i = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\nu}_i + \boldsymbol{\delta}_i \quad KT \times 1$$

$Kp \times 1$ (for $\boldsymbol{\beta}$)

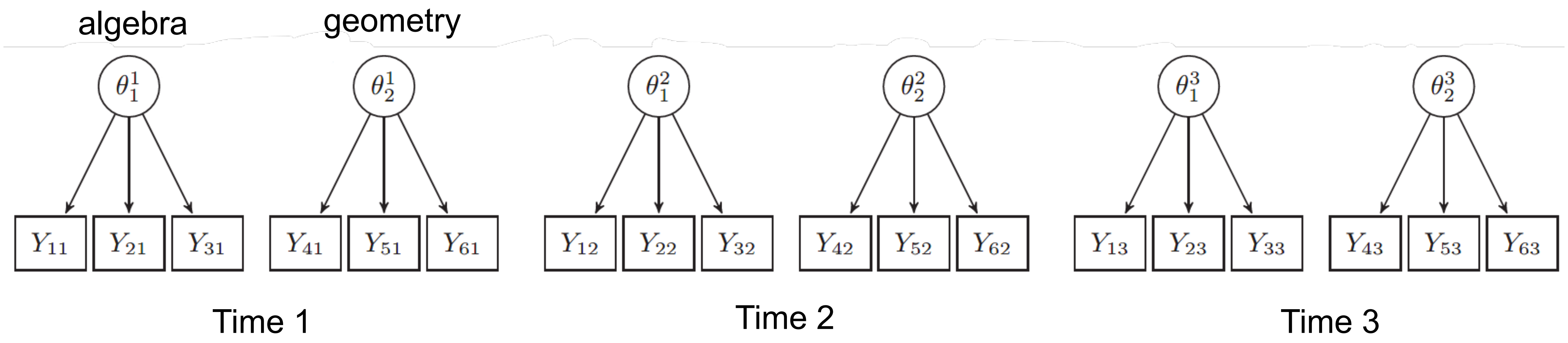
$Kq \times 1$ (for $\boldsymbol{\nu}_i$)

$\sim \text{multinormal}(0, \Sigma_\nu)$ where Σ_ν is a full matrix

K intercepts
K slopes for the first fixed covariate,
K slopes for the second fixed covariate
…

Longitudinal IRT models

- The relationship of the latent traits over time



$$\boldsymbol{\theta}_i = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\nu}_i + \boldsymbol{\delta}_i$$

Example: a simple linear trajectory without any additional covariates

$$\theta_{ik}^t = \pi_{i0k} + \pi_{i1k} \times (t-1) + \delta_{ik}^t$$

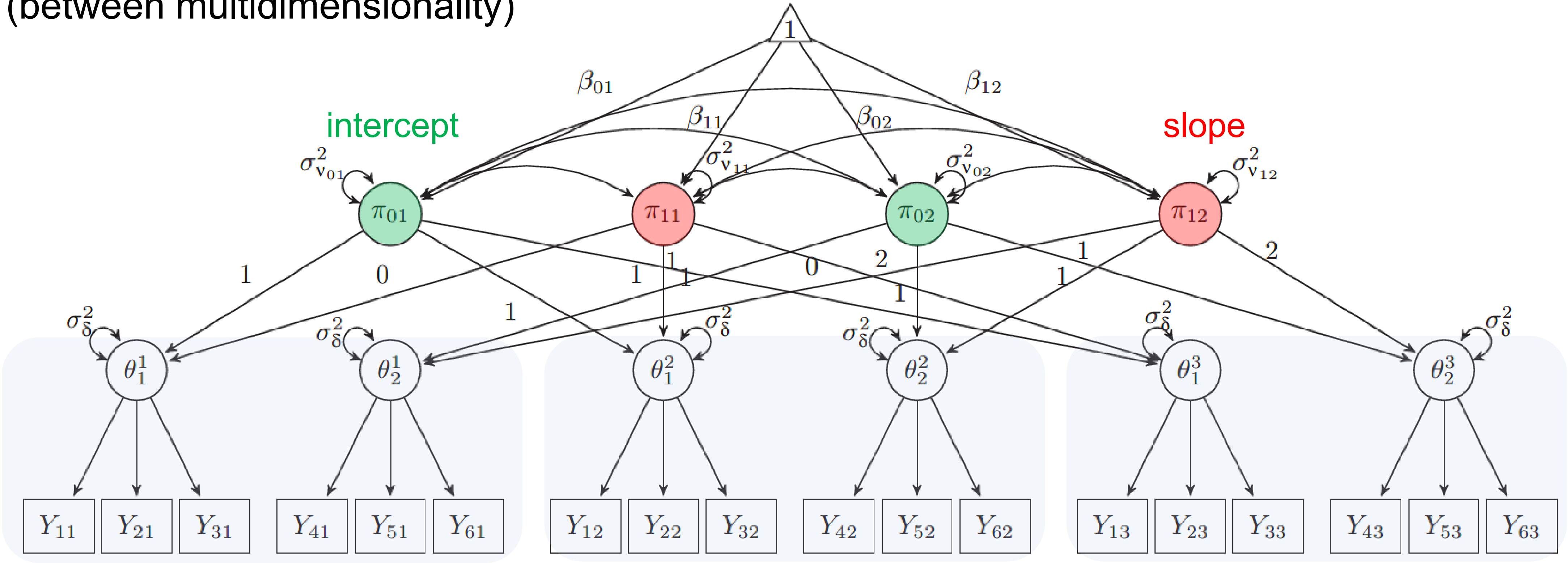
$$\pi_{i0k} = \beta_{0k} + \nu_{i0k}$$

$$\pi_{i1k} = \beta_{1k} + \nu_{i1k}$$

Longitudinal IRT models

- Three time points, two dimensions, three items per domain

  (between multidimensionality)



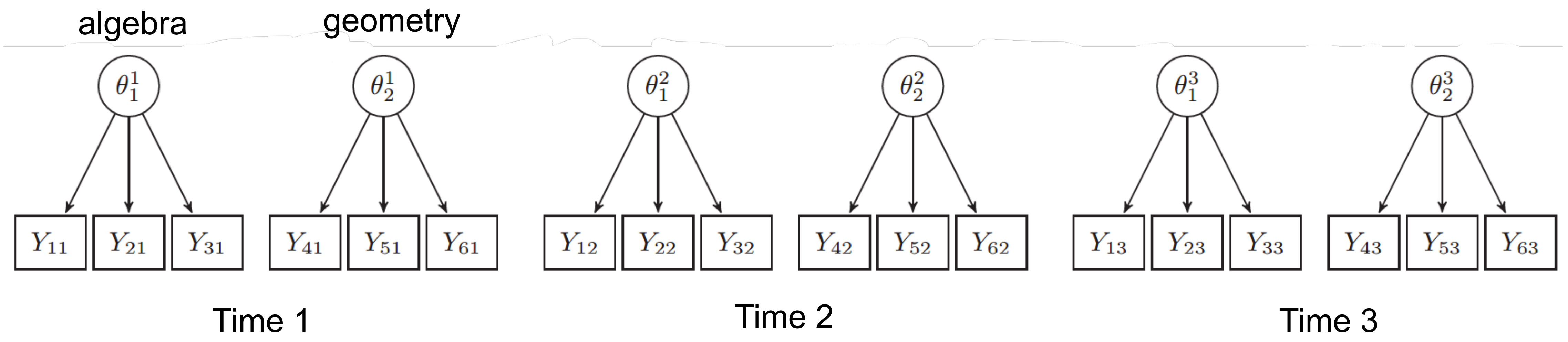
$$\theta^t_{ik} = \pi_{i0k} + \pi_{i1k} \times (t - 1) + \delta^t_{ik}$$

$$\pi_{i0k} = \beta_{0k} + \nu_{i0k}$$

$$\pi_{i1k} = \beta_{1k} + \nu_{i1k}$$

Longitudinal IRT models

- The scale of parameters: linking

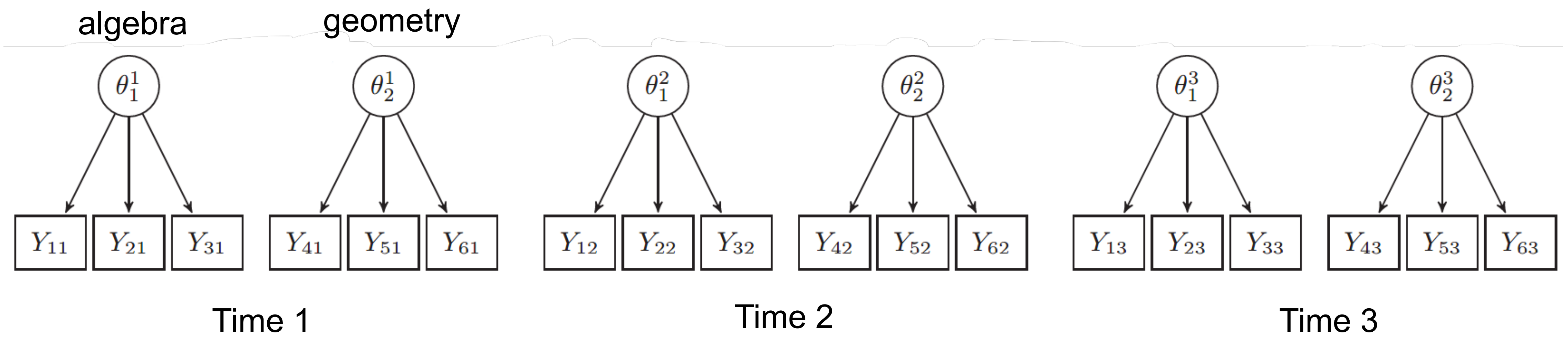


$$\Pr\left(Y_{ij}^t = 1 | \boldsymbol{\theta}_i^t, a_j^t, b_j^t\right) = \frac{1}{1 + \exp\left[-(\mathbf{a}_j^t)^{\mathrm{T}} \boldsymbol{\theta}_i^t + b_j^t\right]}$$

anchor items must still be embedded and load on every domain
→ link the scale of θ_{ik} across time for all $k = 1, \ldots, K$

Longitudinal IRT models

- The scale of parameters: identifiability



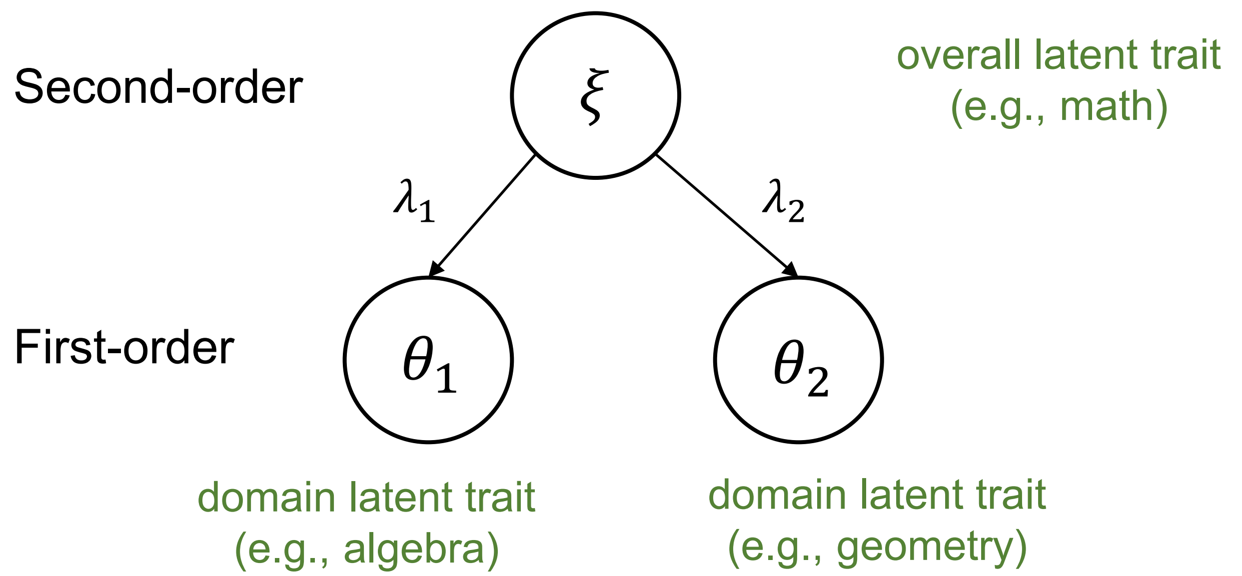
  - If all item parameters are unknown: all constraints are required
  - If anchor items are precalibrated: only constraint 1 is required

$$\theta_{ik}^t = \pi_{i0k} + \pi_{i1k} \times (t-1) + \delta_{ik}^t$$
$$\pi_{i0k} = \beta_{0k} + \nu_{i0k}$$
$$\pi_{i1k} = \beta_{1k} + \nu_{i1k}$$

1. $E(\delta_{ik}^t) = 0$ for all $t = 1, \ldots, T$ and $k = 1, \ldots, K$
2. $\mu_{\pi_{i0k}} = \beta_{0k} = 0$ for all k
   (to fix the mean of θ_{ik}^t at time 1 for all k)
3. $\sigma^2_{\delta_{ik}^1} = c_k$ for all k
   (to fix the variance of θ_{ik}^t at time 1 for all k)

Longitudinal IRT models

- Longitudinal higher-order IRT (L-HO-IRT) model



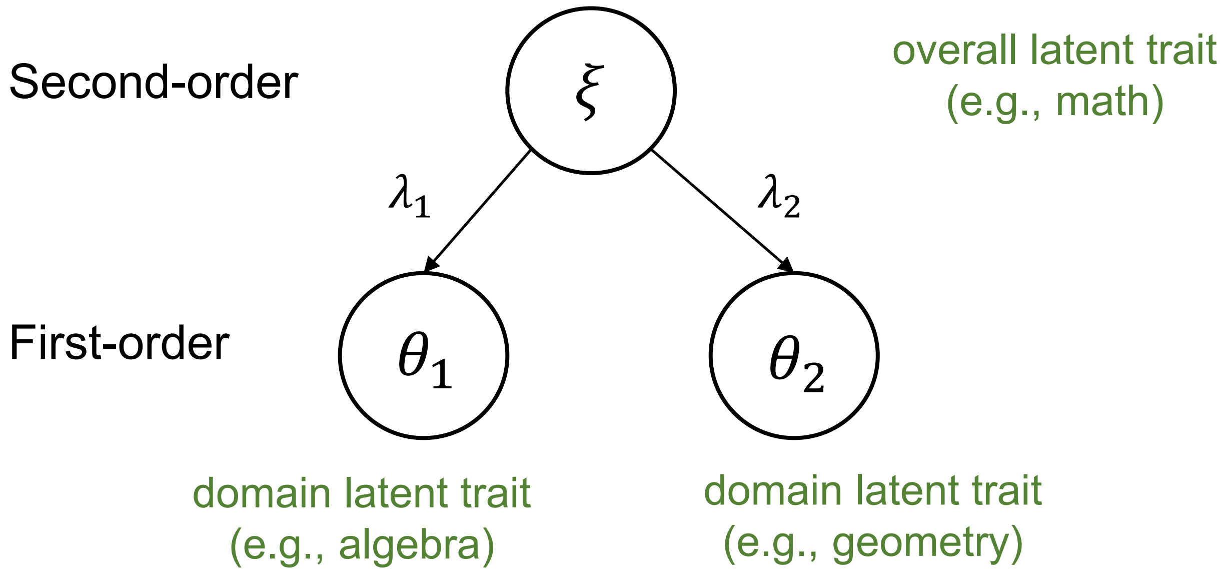
$$\theta_{ik} = \lambda_k \xi_i + \epsilon_{ik}$$

Disturbance term: residual ability not explained by ξ
(uncorrelated across domains → diagonal covariance matrix)

$$\Pr\left(Y_{ij}^t = 1 \middle| \xi_i^t, a_j^t, b_j^t\right) = \frac{1}{1 + \exp\left[-a_j^t\left(\xi_i^t - b_j^t\right)\right]}$$

Longitudinal IRT models

- Longitudinal higher-order IRT (L-HO-IRT) model



$$\theta_{ik} = \lambda_k \xi_i + \epsilon_{ik}$$

The L-HO-IRT model is nested within the L-MIRT model:

1. θ_1 and θ_2 can be uncorrelated in L-MIRT
2. L-HO-IRT restrict the domain-level correlation to be non-zero

Longitudinal IRT models

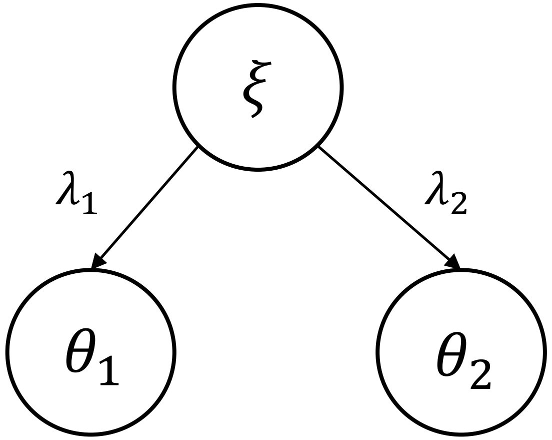
- The relationship of the latent traits over time

$$\theta_{ik} = \lambda_k \xi_i + \epsilon_{ik}$$

$$\boldsymbol{\theta}_i = \boldsymbol{\lambda}(\mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\nu}_i + \boldsymbol{\delta}_i) + \boldsymbol{\epsilon}_i$$

Second-order

First-order



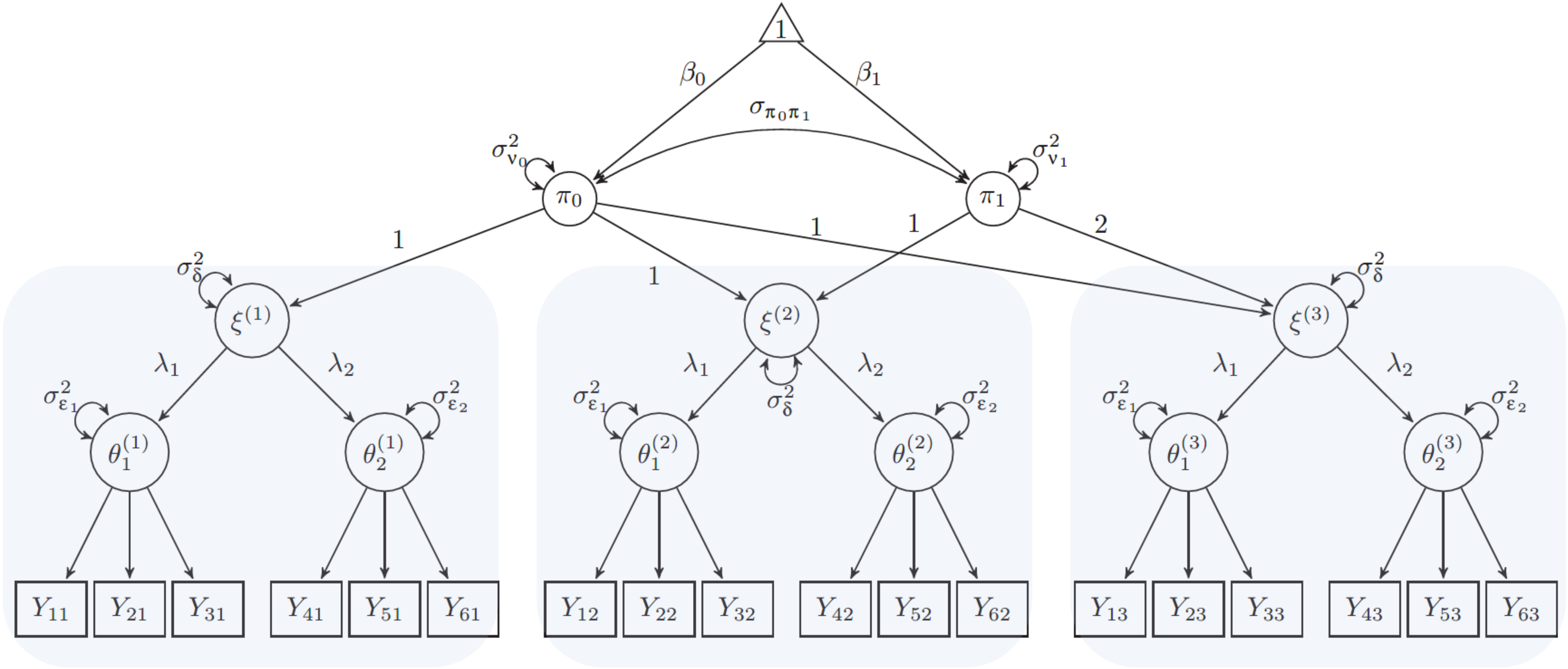
Example: a simple linear trajectory without any additional covariates

$$\begin{aligned}\theta_{ik}^t &= \lambda_k \xi_i^t + \epsilon_{ik}^t \\ &= \lambda_k(\pi_{0i} + \pi_{1i} \times (t-1) + \delta_i^t) + \epsilon_{ik}^t \\ &= \lambda_k \pi_{0i} + \lambda_k \pi_{1i} \times (t-1) + (\lambda_k \delta_i^t + \epsilon_{ik}^t)\end{aligned}$$

the loading remains the same over time
(longitudinal measurement invariance)

Longitudinal IRT models

- Three time points, two domain-specific abilities, three items for each domain

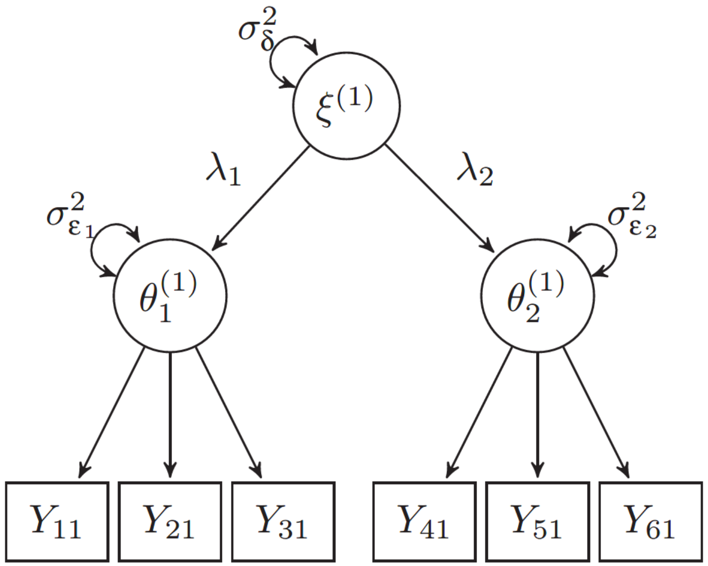


$$\theta_{ik}^t = \lambda_k \xi_i^t + \epsilon_{ik}^t$$
$$= \lambda_k \pi_{0i} + \lambda_k \pi_{1i} \times (t-1) + (\lambda_k \delta_i^t + \epsilon_{ik}^t)$$

Longitudinal IRT models

- The scale of parameters

$$\theta_{ik}^{t} = \lambda_k \xi_i^t + \epsilon_{ik}^{t}$$
$$\xi_i^t = \pi_{0i} + \pi_{1i} \times (t-1) + \delta_i^t$$
$$\pi_{0i} = \beta_0 + v_{0i}$$
$$\pi_{1i} = \beta_1 + v_{1i}$$



1. $E(\delta_{ik}^t) = 0$ for all $t = 1, \dots, T$
2. $\mu_{\pi_{0i}} = \beta_0 = 0$
   (to fix the mean of ξ_i^t at time 1)
3. $E(\epsilon_{ik}^t) = 0$ for all $t = 1, \dots, T$ and $k = 1, \dots, K$
4. Fix the residual variances at time 1: $\sigma_{\epsilon_{ik}^1}^2 = c_k^1$
5. Set one of the loading λ_k for some k to a constant
   (as a reference indicator)

- If all item parameters are unknown: all constraints are required
- If anchor items are precalibrated: only constraint 1,3,5 are required

Model Estimation

- Can be viewed as a multilevel LGC model with the lowest level represented by categorical indicators

→

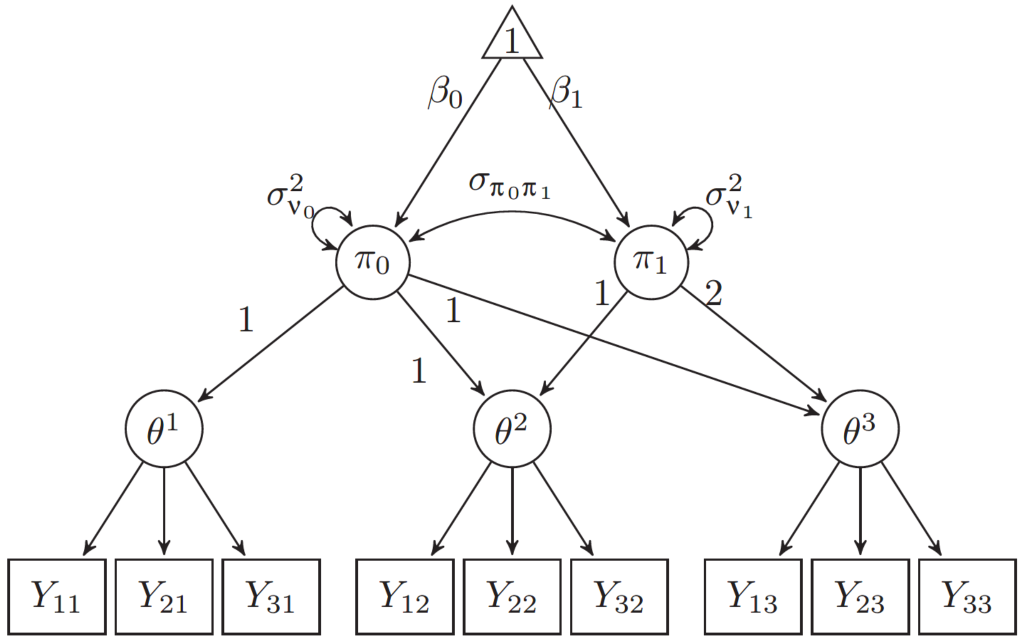
integrate the likelihood over the distribution of random effects

1. Marginal likelihood estimation

```
ANALYSIS: TYPE = GENERAL;
          ESTIMATOR = MLR;
          LINK = LOGIT;
          INTEGRATION = MONTECARLO;
```

2. Markov chain Monte Carlo method

```
ANALYSIS: ESTIMATOR = BAYES;
          CHAINS = 1;
          FBITER = 50000;
          POINT = MEAN;
```



A Real Data Example

- Data
  - math assessments (2009 – 2012): 327 students (grades 3 - 6)
  - five-dimensional, simple structure test with precalibrated item parameters
  - 2009: 57 items (with 23, 9, 7, 11, and 7 items, respectively, measuring each dimension)
  - 2010-2012: 52 items (with 23, 9, 7, 11, and 7 items, respectively, measuring each dimension)
  - precalibrated anchor items were embedded within each of the five dimensions across all 4 years
- Model
  - assume to have only random intercepts and slopes
  - follow two-parameters IRT model
- Estimation
  - MCMC with 30,000 iterations and half of them are discarded as burn-in

Results

0 → intercept
1 → slope

Structural Model Parameter Estimates for Three Different Models

Models	NP	Fixed Effects $(\beta_0\ \ \beta_1)$	Random Effects $\begin{pmatrix}\sigma^2_{\pi_{0i}} & \\ \sigma_{\pi_{0i}\pi_{1i}} & \sigma^2_{\pi_{1i}}\end{pmatrix}$	Others
L-UIRT	275	$(-.653\ \ .472)$	$\begin{pmatrix} .081 & \\ -.008 & .005 \end{pmatrix}$	$\boldsymbol{\sigma}^2_{\delta_i} = (.048\ \ .063\ \ .046\ \ .015)$
L-MIRT	351	$\begin{pmatrix} -.652 & .509 \\ -.633 & .428 \\ -.421 & .356 \\ -.659 & .444 \\ -.795 & .524 \end{pmatrix}$	$\begin{pmatrix} .145 & .014 \\ .255 & .039 \\ .267 & .038 \\ .169 & .042 \\ .120 & .017 \end{pmatrix}$	$\boldsymbol{\sigma}^2_{\delta_i} = \begin{pmatrix} .051 & .087 & .052 & .019 \\ .063 & .066 & .025 & .037 \\ .032 & .048 & .009 & .029 \\ .035 & .055 & .042 & .031 \\ .054 & .010 & .031 & .013 \end{pmatrix}$
L-HO-IRT	299	$(-.702\ \ .514)$	$\begin{pmatrix} .102 & \\ -.009 & .007 \end{pmatrix}$	$\boldsymbol{\lambda} = \begin{pmatrix} 1^* \\ .768 \\ .734 \\ .882 \\ .935 \end{pmatrix}$ $\boldsymbol{\sigma}^2_{\delta_i} = (.052\ \ .071\ \ .061\ \ .015)$ $\boldsymbol{\sigma}^2_{\epsilon_i} = \begin{pmatrix} .028 & .031 & .016 & .017 \\ .121 & .091 & .088 & .044 \\ .018 & .031 & .013 & .011 \\ .059 & .020 & .018 & .034 \\ .059 & .012 & .013 & .008 \end{pmatrix}$

Note. NP denotes the number of free parameters in each model. The covariances between random intercepts and random slopes from the L-MIRT model are omitted to save space because they are between −.01 and .01. "*" denotes a fixed constant. IRT = item response theory; L-UIRT = longitudinal unidimensional IRT; L-MIRT = longitudinal multidimensional IRT; L-HO-IRT = longitudinal higher order IRT.

Results

0 → intercept
1 → slope

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Results

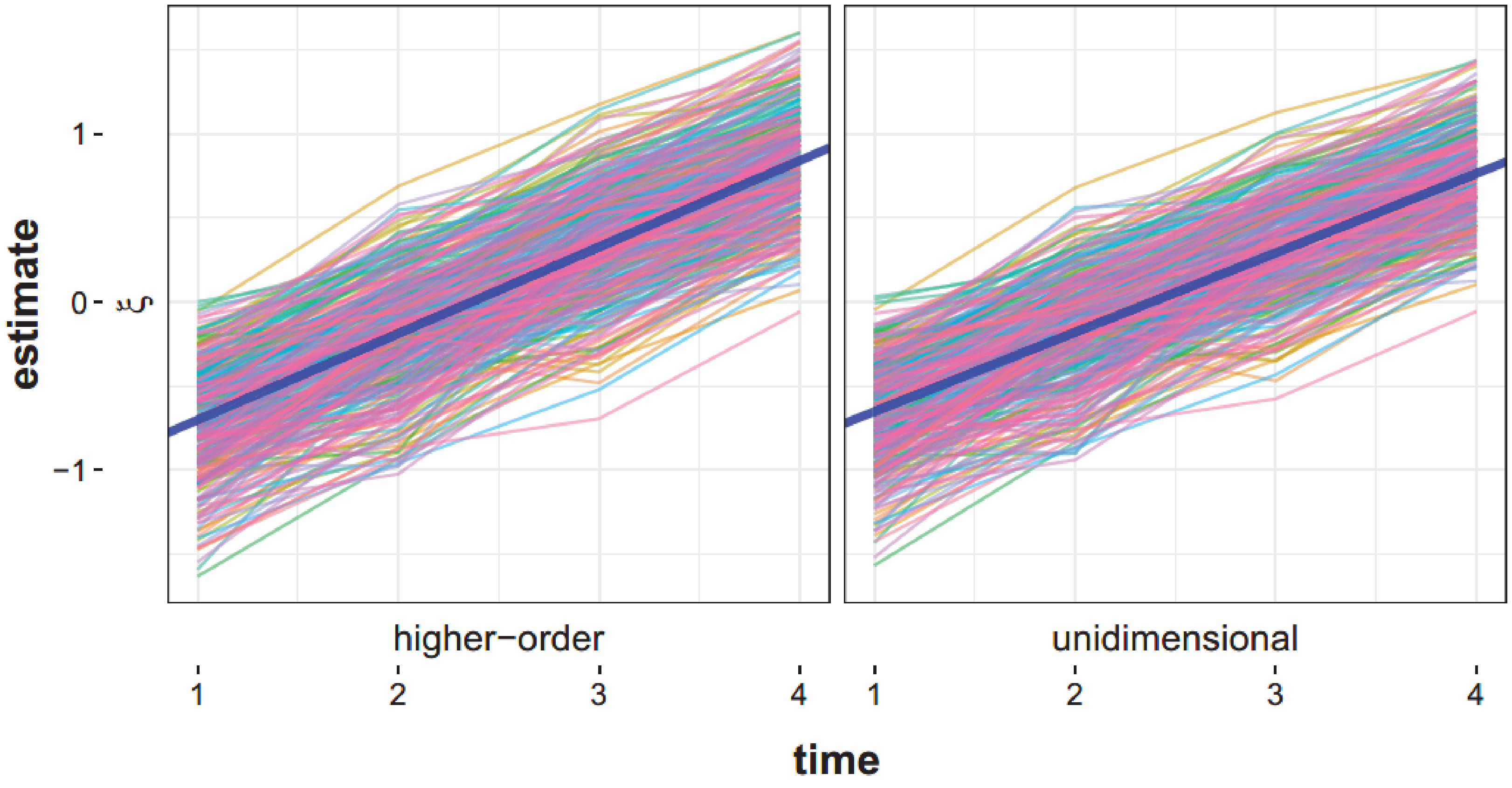

FIGURE 4. *A spaghetti plot, illustrating the linear trend of ξ (overall-level ability) on math between Grades 3 and 6 for* N = *327 students. The left panel is obtained from the longitudinal higher order item response theory model, and the right panel is from the longitudinal unidimensional item response theory model. The bolded, slanted line in the center of the spaghetti depicts the estimated fixed effect of time.*

Results

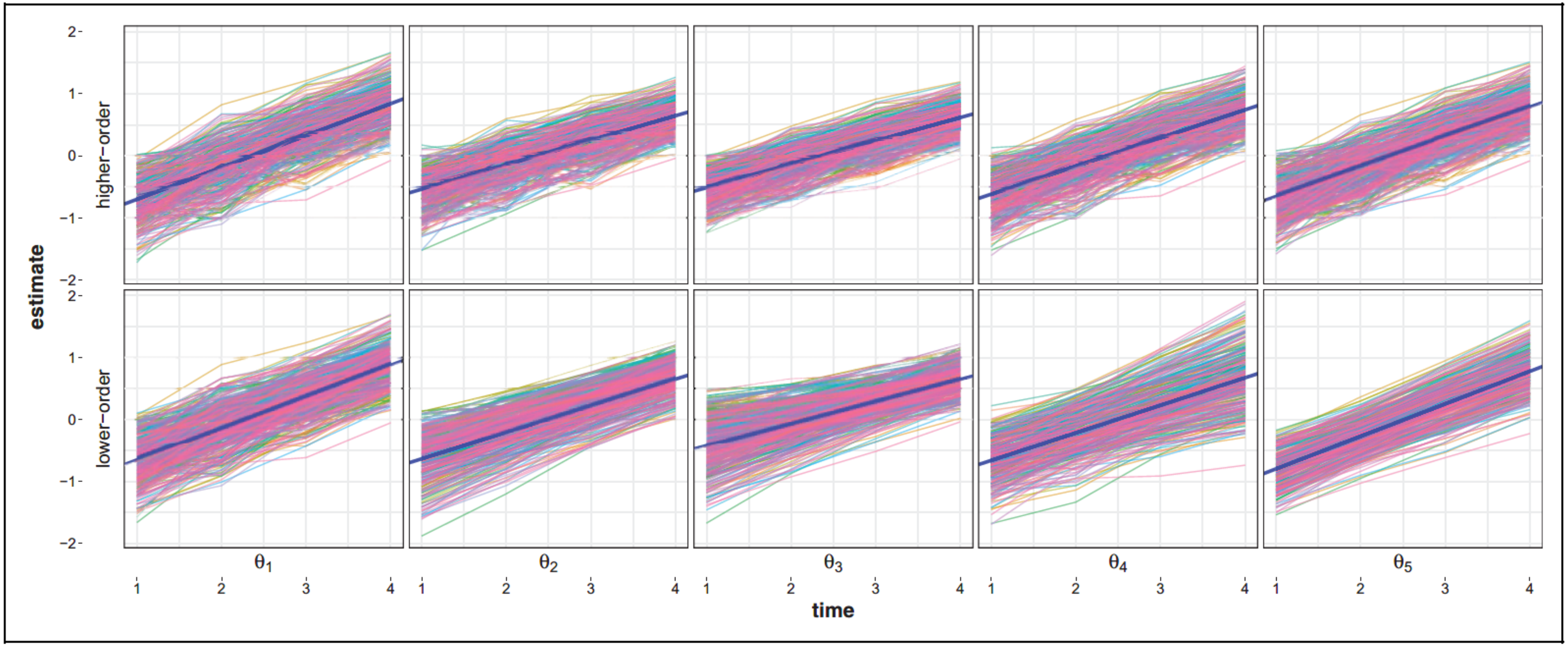

FIGURE 5. *A spaghetti plot, illustrating the linear trend of θ (domain-level ability) on math between Grades 3 and 6 for* N = *327 students. The upper panels are obtained from the longitudinal higher order item response theory model, and the lower panels are from the longitudinal multidimensional item response theory model. The bolded, slanted line in the center of the spaghetti depicts the estimated fixed effect of time.*

Conclusion

- the L-UIRT model is the simplest
- the L-MIRT model describes change in multiple, correlated latent traits
- the L-HO-IRT model (1) simultaneously models the growth trajectories of both overall- and domain-specific abilities and (2) allows for a shift in domain coverage over time

- Future studies can consider how to model nonlinear growth patterns

Thanks for Listening!

Reporter: Yingshi Huang

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